On the Stability of Static Poisson Networks Under Random Access

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***Abstract*— We investigate the stable packet arrival rate region of a discrete-time slotted random access network, where the sources are distributed as a Poisson point process. Each of the sources in the network has a destination at a given distance and a buffer of infinite capacity. The network is assumed to be random but static, i.e., the sources and the destinations are placed randomly and remain static during all the time slots. We employ tools from queueing theory as well as point process theory to study the stability of this system using the concept of dominance. The problem is an instance of the interacting queues problem, further complicated by the Poisson spatial distribution. We obtain sufficient conditions and necessary conditions for stability. Numerical results show that the gap between the sufficient conditions and the necessary conditions is small when the access probability, the density of transmitters, or the SINR threshold is small. The results also reveal that a slight change of the arrival rate may greatly affect the fraction of unstable queues in the network.**

***Index Terms*— Interacting queues, Poisson bipolar model, random access, stability, stochastic geometry.**

I. Introduction

A. Motivation

IN LARGE scale wireless networks, concurrent transmissions lead to interference between terminals. The randomness in the deployment of the transmitters makes accurate modeling and analysis of the interference complicated. Recently, the introduction of the point process theory has provided great convenience for modeling and analyzing the performance of wireless networks [2]–[4]. However, most of the analytical works assume that the terminals are backlogged, i.e., that the terminals always have packets to transmit. In case where each terminal provides a buffer for queueing, the problem becomes more practically relevant and more challenging. For example, a primary problem is to study the stability of the queues in the large scale network. It can be observed from the above description that there are two issues of interest: (a) the random arrival of the packets at the terminals; (b) the noise, the fading, the interference, and the random access protocol that affect the transmission of these packets. The situation is complicated because it involves interacting queues, i.e., the serving rate of each queue depends on the sizes of all the queues. Most of previous works treat these two issues separately. The approaches based on queueing theory focus on the random arrival of the packets but ignore the physical layer as well as the effect of noise and interference [5]–[9]. Other approaches based on the multi-access information theory focus on the physical layer and analyze the transmission process but ignore the random arrival of packets [10]. The approaches based on queueing theory are often used to analyze the performance of scheduling algorithms, whereas the approaches based on the multi-access information theory mostly employ the assumption that all terminals are backlogged, and thus the results obtained constitute upper or lower bounds for the performance of certain schemes. The analysis of interacting queues requires the combination of queueing theory and multi-access information theory and is notoriously difficult.

The analyses of interacting queues have been mostly based on the slotted ALOHA protocol with a simplified physical layer [11]. In most of the works, a discrete-time slotted ALOHA system with N terminals is considered. Each terminal maintains a buffer of infinite capacity to store the incoming packets. The time is divided into discrete slots with equal duration, and in each time slot, each terminal attempts to transmit its head-of-line packet with a certain probability if its buffer is not empty. A collision occurs if two or more terminals transmit simultaneously. When a collision occurs, all terminals involved in the collision retransmit the packet in the next time slot with the same access probability. For this simplified system, the exact stability region was characterized for two [5], [6] and three [7] terminals. When $N > 3$, only sufficient conditions and necessary conditions for stability were obtained [7], [12], [13]. In [14], the stability region of a queueing network with dependent servers, described by the definition

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of subsets that can be activated simultaneously, is studied. In [15], the tradeoff between the capacity and the exact end-to-end queueing delay of cell partitioned networks is analyzed and studied.

In practical wireless networks, the interference between transmissions cannot be accurately modeled as collisions. The interaction among the queues at the transmitters in practical wireless networks is thus more intricate than the aforementioned discrete-time ALOHA system. In this work, we model a large-scale wireless network using the Poisson point process (PPP). Combined with the signal-to-interference-and noise-ratio (SINR) model for successful reception, we explore the effect of random traffic arrival and queueing on the stability of large scale wireless networks.

B. Contributions

We combine queueing theory and stochastic geometry to analyze the stability region of a static Poisson network, in which the transmitters and the receivers are placed randomly at the beginning and then remain static during all the time slots. Compared with high-mobility networks in which the nodes are regenerated independently in each time slot, the static Poisson network is more challenging to analyze since inherent correlations of the interference and signal levels persist among different time slots, due to the common randomness caused by the static locations of the nodes. Most of the practical networks are approximately static because the locations of the terminals cannot drastically change within a short time period, and the statistics obtained by spatially averaging over a large region in static networks are of great significance. From the ergodicity of the PPP, the ensemble averages obtained by averaging over the point process equal the spatial averages obtained by averaging over an arbitrary realization of the PPP over a large region. Intuitively, a direct impact of the static characteristic is that if a transmission fails at a previous time slot, there is an increased probability that it will also fail in the next few time slots [16]. If each transmitter maintains a buffer of infinite capacity to store the packets generated, the network becomes even more complicated to analyze because of the interacting queues problem. We introduce the notion of ϵ-stability, which is a generalization of stability suitable for Poisson networks. By applying the concept of dominance [5], [17], we derive sufficient conditions and necessary conditions for ϵ-stability. Numerical results illustrate the sufficient conditions and necessary conditions and reveal how they vary with system parameters.

C. Related Work

Existing works about interacting queueing systems are mostly based on the discrete-time slotted random access systems in which the transmission fails when two or more terminals transmit in the same slot. Previous analyses have yielded only bounds to the stability regions [5]–[9]. Exact stability regions have been characterized only for cases when the number of terminals is two [5], [6] or three [7]. The stability and delay of multi-access systems with an infinite number of transmitters and with a simplified physical layer were studied in [18]. The work in [19] studied the geometric properties of the stability region of a slotted random access system. All these works considered the collision-based model, and the work in [17] investigated the exact stability region of SINR-based two-user interference channel.

Applications of point process theory to analyze the performance of wireless networks can be found in [2]–[4], [10], [21], and [22]. The method is widely adopted in the literature because it is analytically tractable and reflects the randomness in the practical deployment of wireless network [22], [23]. The works related to static Poisson networks include the analysis of the interference correlation [24], [25] and the local delay, which is defined as the number of time slots required for a node to successfully transmit a packet [26]–[29]. In this line of research, an implicit assumption is that the networks are backlogged. In practice, the packets arrive at each source randomly, and each source maintains a buffer to store the packets. The stability and delay of high-mobility networks were analyzed in [30] using a combination of queueing theory and stochastic geometry. In the high-mobility network, the queue sizes and the serving rates are decoupled; however, practical networks are mostly static at the time scale of the transmissions, and the decoupling exploited in high-mobility networks does not apply.

The remaining part of the paper is organized as follows. Section II describes the spatial distribution model, the arrival process, and the access protocol. Section III gives the definition of stability. Based on the concept of dominance and some related relaxations, Section IV and Section V establish sufficient conditions and necessary conditions for stability. Section VI analyzes provides asymptotic behaviors and numerical results. Finally, Section VII concludes the paper.

II. System Model

In order to analyze the stability of a large scale network, we adopt a simple yet general model. We consider a discrete-time slotted random access system with transmitters and receivers distributed as a Poisson bipolar network [3, Definition 5.8], i.e., we model the locations of the transmitters as a PPP $\Phi = \{x_i\} \subset \mathbb{R}^d$ of intensity λ. Each transmitter is paired with a receiver at a fixed distance r_0 and a random orientation. In the analysis, we will condition on $x_0 \in \Phi$ at which the typical transmitter under consideration is located, where $r_0 = |x_0|$ is the distance of this point to the origin at which the corresponding receiver is located (see Fig. 1). The time is divided into discrete slots with equal duration, and each transmission attempt occupies one time slot. We assume that the network is static, i.e., the locations of the transmitters and the receivers are generated once at the beginning and then kept unchanged during all time slots.

We use the Rayleigh block fading model in which the power fading coefficients remain static over each time slot, and are spatially and temporally independent with exponential distribution of mean 1. Let α be the path loss exponent and $h_{k,x}$ be the fading coefficient between transmitter x and the considered receiver located at the origin o in time slot k. All transmitters are assumed to transmit at unit power. The power of the thermal noise is set as W. We assume an SINR

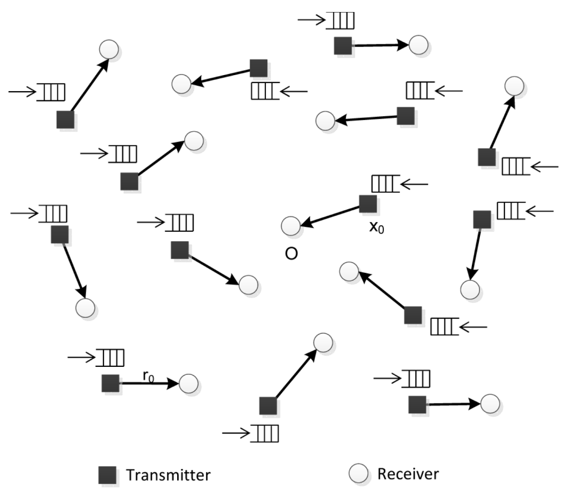

Fig. 1. A snapshot of the Poisson bipolar network with random access.

threshold model: if the SINR over a link is above a threshold θ, the link can be successfully used for information transmission at spectral efficiency $\log_2(1+\theta)$ bits/second/Hz.

Each transmitter has a buffer of infinite capacity to store the packets generated. Each transmitter generates packets according to a Bernoulli process with arrival rate ξ $(0 \le \xi \le 1)$ packets per time slot, i.e., ξ is the probability of an arrival in any given time slot. In fact, the analysis and derivations in the following are not limited to the discrete Bernoulli arrival of packets since the time is slotted. For example, if the packets obey Poisson arrival, they will be served at the beginning of the next time slot if there is no packet waiting in the queue. The arrival processes of different transmitters are independent. In each time slot, each transmitter attempts to send its head-of-line packet with probability p if its buffer is not empty. We assume that the feedback of the status of each attempt of transmission, either successful or failed, is instantaneous so that each transmitter is aware of the outcome. If a transmission attempt fails, the transmitter retransmits the packet in the next time slot with probability p; on the other hand, if a transmission attempt is successful, the transmitter removes the packet from the buffer.

For any time slot $k \in \mathbb{N}^+$, let Φ_k be the set of transmitters that are transmitting in that time slot. The interference at the typical receiver located at the origin o in time slot k is

$$I_k = \sum_{x\in\Phi\setminus\{x_0\}} h_{k,x}|x|^{-\alpha}\mathbf{1}(x\in\Phi_k). \tag{1}$$

When the typical transmitter is active, the SINR of the typical receiver in time slot k is

$$\mathrm{SINR}_k = \frac{h_{k,x_0} r_0^{-\alpha}}{W + \sum_{x\in\Phi\setminus\{x_0\}} h_{k,x}|x|^{-\alpha}\mathbf{1}(x\in\Phi_k)}. \tag{2}$$

In the proposed network model, each transmitter maintains a queue with Bernoulli arrival. However, since the realization of the PPP is irregular, the distances to the interferers are different from the perspectives of individual receivers. Therefore, there are always some transmitters that experience poor performance (i.e., low success probability) while some others that experience good performance (i.e., high success probability). In view of this, even with the same arrival rate for all transmitters in a large scale network, the queues of the transmitters experiencing poor performance may become unstable because of the low success probability. Therefore, the characterization of the stability region of such networks is important and challenging.

Since we condition on Φ having a point at x_0, the relevant probability measure of the point process is the Palm probability $\mathbb{P}^{x_0}$. Correspondingly, the expectation, denoted by $\mathbb{E}^{x_0}$, is taken with respect to the measure $\mathbb{P}^{x_0}$. Whether the transmission of the typical transmitter x_0 is successful or not is uncertain, and the randomness comes from four aspects: the realization of PPP, the random access, the fading and the random arrival of traffic. Let C_Φ^k be the event that the transmission of the typical transmitter x_0 succeeds in time slot k conditioned on the PPP Φ. C_Φ^k is the intersection of two events: that the transmission is scheduled by the random access scheme and that the scheduled transmission is successful. Let $\mathbb{P}^{x_0}(C_\Phi^k) = \mathbb{P}(\mathrm{SINR}_k > \theta \mid \Phi, x_0 \in \Phi)$ be the success probability of the transmission of the typical transmitter x_0 in time slot k conditioned on the PPP Φ. $\mathbb{P}^{x_0}(C_\Phi^k)$ varies with the index k because the empty or non-empty status of the queues at the interferers change over time, resulting in interference variation. In the following discussions, we will show how the stability depends on the statistical properties of $\mathbb{P}^{x_0}(C_\Phi^k)$.

III. Notion of ε-Stability

For an isolated transmitter, by the Loynes theorem [31], if the arrival process and the serving process are stationary, the sufficient and necessary condition for stability is that the average service rate is larger than the average arrival rate. However, strict stability (i.e., all queues are stable) for a large scale network is not achievable (except for the trivial case of $\xi = 0$) since there always exist some transmitters whose queues are unstable in the static Poisson network. Thus, we introduce the notion of ε-stability as follows.

Definition 1: For any $0 \le \varepsilon \le 1$*, the* ε*-stability region* $\mathcal{S}_\varepsilon$ *is defined as*

$$\mathcal{S}_\varepsilon \triangleq \left\{\xi \in \mathbb{R}^+ : \mathbb{P}^{x_0}\left\{\lim_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\mathbb{P}^{x_0}(C_\Phi^k) \le \xi\right\} \le \varepsilon\right\}. \tag{3}$$

Definition 2: The supremum of the ε*-stability region* $\mathcal{S}_\varepsilon$*, i.e.,* $\xi_{\mathrm{c},\varepsilon} \triangleq \sup \mathcal{S}_\varepsilon$*, is called the critical arrival rate. The network is* ε*-stable if and only if* $\xi \le \xi_{\mathrm{c},\varepsilon}$.

Remark 1: $\mathbb{P}^{x_0}\{\lim_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\mathbb{P}^{x_0}(C_\Phi^k) \le \xi\}$ *is the probability that the queue at the typical transmitter is unstable. We declare the network to be* ε*-stable if the probability for the queue at the typical transmitter being unstable is less than a threshold* ε $(0 < \varepsilon < 1)$. *The probability for the queue at the typical transmitter being unstable is obtained by averaging over the point process. Based on the ergodicity of the PPP, i.e., the ensemble averages obtained by averaging over the point process equal the spatial averages obtained by averaging an arbitrary realization of the PPP over a large region, the probability that the queue at the typical transmitter is unstable equals the proportion of unstable transmitters in* *the network. Thus, a network is ε-stable implies that the proportion of unstable transmitters in the network is less than ε.*

Deriving the ε-stability region $\mathcal{S}_\varepsilon$ is equivalent to obtaining the critical arrival rate $\xi_{c,\varepsilon}$, which is rather difficult because of the interacting queueing problem. Therefore, in the following, we obtain $\xi^{s}_{c,\varepsilon}$ and $\xi^{n}_{c,\varepsilon}$ with $\xi^{s}_{c,\varepsilon} \le \xi_{c,\varepsilon} \le \xi^{n}_{c,\varepsilon}$. Then, $\xi \le \xi^{s}_{c,\varepsilon}$ and $\xi \le \xi^{n}_{c,\varepsilon}$ correspond to a sufficient condition and a necessary condition for ε-stability, respectively.

For example, consider a very simple system that consists of only the typical transmission, i.e., the interference from other transmitters in the system are ignored. The success probability for that typical transmitter is $p\exp\left(-W\theta r_0^\alpha\right)$. By applying the Loynes theorem, we get the condition for stability of the queue at the typical transmitter ξ_0 as

$$\xi \le \xi_0 \triangleq p\exp\left(-W\theta r_0^\alpha\right). \tag{4}$$

In fact, all the sufficient conditions and necessary conditions in the following sections can be expressed in the form of $\xi \le \beta\xi_0$ with $0 \le \beta \le 1$, where ξ_0 captures the effect of noise and random access at the typical transmission while β captures the effect of the interference which is affected by the random access at the interfering links.

IV. Sufficient Conditions

In order to derive sufficient conditions for ε-stability, we consider a dominant system [5], [9], [17]. In the dominant system the typical transmitter behaves exactly the same as in the original system. However, for the other transmitters in the dominant system, when the queue at a transmitter becomes empty, it continues to transmit "dummy" packets with the access probability p, thus continuing to cause interference to other transmissions with probability p. So in the dominant system, the queue size at each transmitter is always no smaller than that in the original system, provided the queues start with the same initial conditions. In the dominant system, the success probability given Φ is the same for different time slots because all transmitters always have packets to transmit, and the fading and the scheduling result of random access are i.i.d. between different time slots. The ε-stability region $\mathcal{S}_\varepsilon$ is simplified into $\overline{\mathcal{S}}_\varepsilon = \left\{\xi \in \mathbb{R}^+ : \mathbb{P}^{x_0}\left\{\mathbb{P}^{x_0}(\mathcal{C}_\Phi) \le \xi\right\} \le \varepsilon\right\}$. By deriving the ε-stability conditions for the dominant system, we get a sufficient condition for the original system to be ε-stable.

Theorem 1: Given a slotted random access system with the transmitters distributed as a PPP and with Bernoulli packet arrivals, a sufficient condition for the system to be ε-stable is

$$\xi \le \xi^{s}_{c,\varepsilon}, \tag{5}$$

where $\xi^{s}_{c,\varepsilon} \triangleq \sup\overline{\mathcal{S}}_\varepsilon$ is given by

$$\xi^{s}_{c,\varepsilon} = \sup\left\{\xi \in \mathbb{R}^+ : \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{1}{\omega} \operatorname{Im}\left\{\left(\frac{\xi_0}{\xi}\right)^{j\omega} e^{-j\omega C_\delta {}_2F_1(1-j\omega,\,1-\delta;\,2;\,p)}\right\}\mathrm{d}\omega \le \varepsilon\right\}, \tag{6}$$

with $\delta = 2/\alpha$, $C_\delta = p\lambda\pi r_0^2\theta^\delta\Gamma(1+\delta)\Gamma(1-\delta)$, and ${}_2F_1(a,b;c;z)$ is the Gaussian hypergeometric function. Thus, a lower bound on the critical arrival rate $\xi_{c,\varepsilon}$ is $\xi^{s}_{c,\varepsilon}$, i.e., $\xi_{c,\varepsilon} \ge \xi^{s}_{c,\varepsilon}$.

Proof: See Appendix A. □

Remark 2: $\xi^{s}_{c,\varepsilon}$ given by (6) can be written as

$$\xi^{s}_{c,\varepsilon} = \xi_0 \sup\left\{\beta \in \mathbb{R}^+ : \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{1}{\omega} \operatorname{Im}\left\{\beta^{-j\omega} e^{-j\omega C_\delta {}_2F_1(1-j\omega,\,1-\delta;\,2;\,p)}\right\}\mathrm{d}\omega \le \varepsilon\right\}, \tag{7}$$

where ξ_0/ξ in (6) is replaced by $1/\beta$, and β is the parameter introduced after (4) that captures the effect of the interference.

As $\lambda \to 0$, we have $C_\delta \to 0$, and (7) becomes

$$\xi^{s}_{c,\varepsilon} = \xi_0 \sup\left\{\beta \in \mathbb{R}^+ : \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \frac{\sin(\omega\ln\beta)}{\omega}\mathrm{d}\omega \le \varepsilon\right\}. \tag{8}$$

Since $\int_{-\infty}^0 \frac{\sin(\pi x)}{\pi x}\mathrm{d}x = \int_0^\infty \frac{\sin(\pi x)}{\pi x}\mathrm{d}x = \frac{1}{2}$, when $\ln\beta > 0$, the expression $\frac{1}{2} + \frac{1}{\pi}\int_0^\infty \frac{1}{\omega}\sin(\omega\ln\beta)\mathrm{d}\omega$ evaluates to 1; otherwise when $\ln\beta < 0$ it evaluates to 0. Thus, we have

$$\begin{aligned}\xi^{s}_{c,\varepsilon} &= \xi_0 \sup\left\{\beta \in \mathbb{R}^+ : \mathbf{1}\left(\ln\beta > 0\right) \le \varepsilon\right\}\\ &= \xi_0,\end{aligned} \tag{9}$$

where $\mathbf{1}(\cdot)$ is the indicator function. This is exactly the case where the interference is ignored and only noise and fading affect the transmission. Combined with the necessary condition (4) $\xi \le \xi_0$, we get the exact critical arrival rate $\xi_{c,\varepsilon} = \xi_0$ as $\lambda \to 0$.

As $\theta \to 0$, the sufficient condition (7) becomes

$$\begin{aligned}\xi^{s}_{c,\varepsilon} &= \sup\left\{\xi \in \mathbb{R}^+ : \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{\sin\left(\omega\ln p - \omega\ln\xi\right)}{\omega}\mathrm{d}\omega \le \varepsilon\right\}\\ &= p.\end{aligned} \tag{10}$$

As $\theta \to 0$, the necessary condition (4) becomes $\xi \le p$. Thus, we get the exact critical arrival rate $\xi_{c,\varepsilon} = p$ as $\theta \to 0$.

In the original system where the queues interact with each other, if $\lambda \to 0$, the interference is negligible, and a transmission in the network is only affected by the thermal noise. And if $\theta \to 0$, a transmission is almost surely successful if it is scheduled. Therefore, in these cases, the serving processes of the packets at different transmitters can be approximated as decoupled and independent, and the critical arrival rates above are intuitively reasonable.

The following corollary gives a closed-form sufficient condition that is weaker than the one given by Theorem 1 but easier to evaluate.

Corollary 1: Given a slotted random access system with the transmitters distributed as a PPP and with Bernoulli arrivals, a sufficient condition for the system to be ε-stable is

$$\xi \le \widetilde{\xi}^{s}_{c,\varepsilon}, \tag{11}$$

where $\widetilde{\xi}^{s}_{c,\varepsilon} \triangleq \max_{n\in\mathbb{N}^+}\eta(n)$, and

$$\eta(n) = \xi_0\varepsilon^{\frac{1}{n}}\exp\left(-\pi\lambda\delta(1-p)^\delta\theta^\delta r_0^2 \sum_{i=1}^{n}\left((1-p)^{-i}-1\right)\frac{\Gamma(i-\delta)\Gamma(n-i+\delta)}{\Gamma(i+1)\Gamma(n-i+1)}\right). \tag{12}$$

Thus, a closed-form lower bound on the critical arrival rate $\xi_{c,\varepsilon}$ is $\widetilde{\xi}^{s}_{c,\varepsilon}$, i.e., $\xi_{c,\varepsilon} \geq \widetilde{\xi}^{s}_{c,\varepsilon}$.

Proof: For all $n \in \mathbb{N}^+$, the cdf of $\mathbb{P}^{x_0}(\mathcal{C}_\Phi)$ is

$$\mathbb{P}^{x_0}\left\{\mathbb{P}^{x_0}(\mathcal{C}_\Phi) \leq \xi\right\} = \mathbb{P}^{x_0}\left\{e^{-n\ln(\mathbb{P}^{x_0}(\mathcal{C}_\Phi))} \geq e^{-n\ln\xi}\right\}. \quad (13)$$

By applying the Markov inequality, we obtain

$$\begin{aligned}
\mathbb{P}^{x_0}&\left\{\mathbb{P}^{x_0}(\mathcal{C}_\Phi) < \xi\right\} < \frac{1}{e^{-n\ln\xi}}\mathbb{E}\left[e^{-n\ln(\mathbb{P}^{x_0}(\mathcal{C}_\Phi))}\right] \\
&= p^{-n}\exp\left(n\ln\xi + n\theta r_0^\alpha W\right) \\
&\quad\times\mathbb{E}\Big[\prod_{x\in\Phi\setminus\{x_0\}}\Big(\frac{p}{1+\theta r_0^\alpha|x|^{-\alpha}}+1-p\Big)^{-n}\Big] \\
&= \Big(\frac{\xi}{\xi_0}\Big)^n\exp\Big(-2\pi\lambda \\
&\quad\times\int_0^\infty\Big(1-\Big(\frac{p}{1+\theta r_0^\alpha r^{-\alpha}}+1-p\Big)^{-n}\Big)r\mathrm{d}r\Big) \\
&= \Big(\frac{\xi}{\xi_0}\Big)^n\exp\Big(2\pi\lambda \\
&\quad\times\int_0^\infty\frac{(1+\theta r_0^\alpha r^{-\alpha})^n-(1+(1-p)\theta r_0^\alpha r^{-\alpha})^n}{(1+(1-p)\theta r_0^\alpha r^{-\alpha})^n}r\mathrm{d}r\Big). \\
&\overset{(a)}{=} \Big(\frac{\xi}{\xi_0}\Big)^n\exp\Big(2\pi\lambda\sum_{i=0}^n C_n^i(1-(1-p)^i) \\
&\quad\times\int_0^\infty\frac{(\theta r_0^\alpha r^{-\alpha})^i r}{(1+(1-p)\theta r_0^\alpha r^{-\alpha})^n}\mathrm{d}r\Big) \\
&\overset{(b)}{=} \Big(\frac{\xi}{\xi_0}\Big)^n\exp\Big(\pi\lambda n\delta(1-p)^\delta\theta^\delta r_0^2 \\
&\quad\times\sum_{i=1}^n((1-p)^{-i}-1)\frac{\Gamma(i-\delta)\Gamma(n-i+\delta)}{\Gamma(i+1)\Gamma(n-i+1)}\Big).
\end{aligned} \quad (14)$$

where $C_n^i = n!/(i!(n-i)!) = \Gamma(n+1)/(\Gamma(i+1)\Gamma(n-i+1))$ is the binomial coefficient. (a) holds from the binomial expansion and the exchange of summation and integral. (b) follows from the relationship between the beta function $B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\mathrm{d}t$ and the gamma function and from the fact that the term for $i=0$ equals zero.

Since the above inequality holds for all $n \in \mathbb{N}^+$, we have

$$\begin{aligned}
\overline{\mathcal{S}}_\varepsilon \supset \bigcup_{n\in\mathbb{N}^+}\Big\{\xi\in\mathbb{R}^+ : \Big(\frac{\xi}{\xi_0}\Big)^n \\
\times\exp\Big(\pi\lambda n\delta(1-p)^\delta\theta^\delta r_0^2\sum_{i=1}^n((1-p)^{-i}-1) \\
\times\frac{\Gamma(i-\delta)\Gamma(n-i+\delta)}{\Gamma(i+1)\Gamma(n-i+1)}\Big) \leq \varepsilon\Big\}.
\end{aligned} \quad (15)$$

Taking the supremum on both sides of (15) results in

$$\sup\overline{\mathcal{S}}_\varepsilon > \max_{n\in\mathbb{N}^+}\eta(n), \quad (16)$$

where $\eta(n)$ is given by (12). Letting $\widetilde{\xi}^{s}_{c,\varepsilon} = \max_{n\in\mathbb{N}^+}\eta(n)$, we get $\widetilde{\xi}^{s}_{c,\varepsilon} < \sup\overline{\mathcal{S}}_\varepsilon = \xi^{s}_{c,\varepsilon}$, indicating that $\xi \leq \widetilde{\xi}^{s}_{c,\varepsilon}$ is also a sufficient condition for ε-stability which is "looser" than $\xi \leq \xi^{s}_{c,\varepsilon}$. □

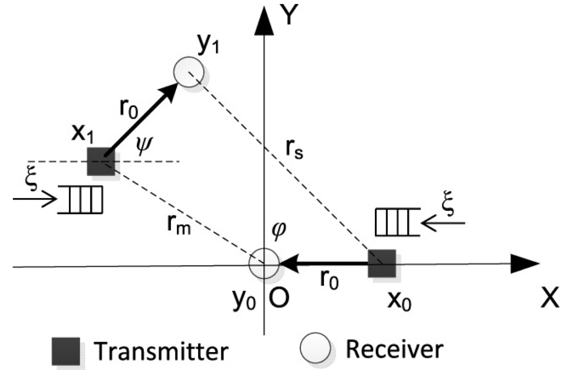

Fig. 2. The simplified system which consists of two pairs of transceivers, i.e., the typical transmission and the nearest interfering transmission in the original system.

V. Necessary Conditions

The simple condition given in (4) is weak because it ignores the interference. In the following, we propose two approaches to derive two different types of necessary conditions for ε-stability. In the derivation of the *type I necessary conditions*, we consider a simplified system in which only the effect of the nearest interferer is considered. Since the interference is reduced in the simplified system, a necessary condition for the typical transmitter to be stable in the original system is that it is stable in the simplified system. In the derivation of the *type II necessary conditions*, we consider a modified favorable system that drops the packets in the interfering transmitters that are not scheduled by the random access or whose transmission failed. Since the interference is not larger than that in the original system and the packets will not accumulate at the interfering transmitters, the ε-stability region will be a subset of that of the original system.

A. Type I Necessary Conditions

First we derive type I necessary conditions and consider a simplified version of the original system, in which only two pairs of transmitters and receivers are considered. One pair is the typical pair in the original system, whose transmitter is located at $x_0 = (r_0, 0)$ and the corresponding receiver y_0 is located at the origin o. The other pair is the pair containing the nearest interferer. Let $x_1 = (r_\mathrm{m}\cos\varphi, r_\mathrm{m}\sin\varphi)$ be the location of the nearest transmitter, where r_m is the distance from the origin and φ is the angle. Let $y_1 = (r_\mathrm{m}\cos\varphi + r_0\cos\psi, r_\mathrm{m}\sin\varphi + r_0\sin\psi)$ be the location of the associated receiver, where ψ is the angle between x_1 and y_1 (see Fig. 2). φ and ψ are independent uniformly distributed random variables in $[0, 2\pi]$. The pdf of r_m is

$$f_{r_\mathrm{m}}(r) = 2\pi\lambda r\exp\left(-\pi\lambda r^2\right). \quad (17)$$

A necessary condition for the original system to be ε-stable is that the probability of the transmitter located at x_0 in the simplified system being unstable is less than ε. Notice that we only need to consider the stability of the queue at the transmitter x_0, i.e., it does not matter whether the interfering transmitter's queue is stable or not. Since r_m is a random variable, it is uncertain whether the queue at the transmitter x_0 is stable or not. However, if r_{m} is given, the stability of the queue at the transmitter x_0 is determined. Therefore, we first derive a sufficient and necessary condition for the transmitter x_0 to be stable when r_{m} is given.

Consider a dominant system of the simplified system, i.e., the transmitter x_0 still transmits "dummy" packets when its queue is empty, thus it keeps causing interference. Unlike the transmitter x_0, the nearest interfering transmitter x_1 in the dominant system behaves the same as in the original simplified system. In fact, a sufficient and necessary condition for the transmitter x_0 in the simplified system to be stable is that it is stable in the dominant simplified system. The sufficiency claims that if the queue at x_0 is stable in the dominant simplified system, then it will be stable in the original simplified system. This is because the interference in the dominant simplified system is larger than that in the original simplified system, resulting in smaller success probability. The necessity claims that if the queue at x_0 is unstable in the dominant simplified system, then it will be unstable in the original simplified system. This is because when the queue at the transmitter x_0 in the dominant simplified system is unstable, the queue size will grow to infinity without emptying with non-zero probability. Notice that as long as the queue at x_0 is not empty, the dominant simplified system and the original simplified system behave identically if starting from the same initial condition, and the dominant simplified system is indistinguishable from the original simplified system under saturation. Thus the sample paths that do not visit queue size zero in the dominant simplified system also occur in the original simplified system, and they constitute a positive measure of all sample paths. Therefore, the queue at x_0 in the original simplified system is also unstable. Combining the two parts, the proof of the sufficiency and the necessity is complete. Therefore, we only need to derive the sufficient and necessary condition for the transmitter x_0 to be stable in the dominant simplified system. Based on these ideas, we get the following lemma.

Lemma 1: For the simplified system with given $\varphi, \psi, r_{\mathrm{m}}$, the sufficient and necessary condition for the queue at the transmitter x_0 to be stable is

$$\xi \leq \begin{cases} \xi_0 \dfrac{(1+(1-p)\theta_{\mathrm{s}})(1+\theta_{\mathrm{m}})}{p(\theta_{\mathrm{m}}-\theta_{\mathrm{s}})+(1+\theta_{\mathrm{s}})(1+\theta_{\mathrm{m}})} & \text{if } r > r_{\mathrm{m}} \\ \xi_0 \dfrac{1+(1-p)\theta_{\mathrm{m}}}{1+\theta_{\mathrm{m}}} & \text{if } r_{\mathrm{s}} \leq r_{\mathrm{m}} \end{cases} \tag{18}$$

where $\theta_{\mathrm{s}} = \theta r_0^{\alpha} r_{\mathrm{s}}^{-\alpha}$, $\theta_{\mathrm{m}} = \theta r_0^{\alpha} r_{\mathrm{m}}^{-\alpha}$ *and* $r_{\mathrm{s}} = \sqrt{(r_{\mathrm{m}}\cos\varphi + r_0\cos\psi - r_0)^2 + (r_{\mathrm{m}}\sin\varphi + r_0\sin\psi)^2}$.

Proof: See Appendix B. □

Lemma 1 gives the sufficient and necessary condition for the queue at the transmitter x_0 to be stable with given $\varphi, \psi, r_{\mathrm{m}}$. For the nearest interferer, $\varphi, \psi, r_{\mathrm{m}}$ are random variables. By applying the results in Lemma 1, we get the following theorem.

Theorem 2: Given a slotted random access system with the transmitters distributed as a PPP and with Bernoulli packet arrivals, a type I necessary condition for the system to be ε-stable is

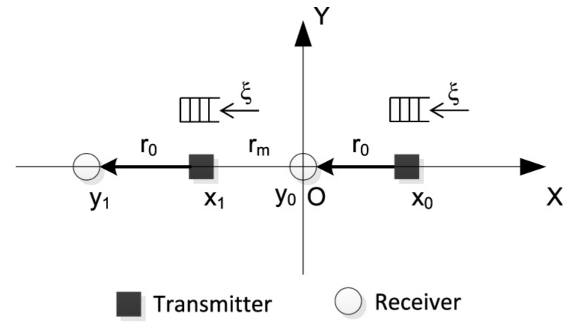

Fig. 3. The simplified system when $\varphi = \psi = -\pi$ which consists of two pairs of transceivers.

$$\xi \leq \xi_{\mathrm{c},\varepsilon}^{\mathrm{n1}} \triangleq \xi_0 \left(1 - \frac{\theta p}{\theta + \left(F_Z^{-1}(\varepsilon)\right)^{\alpha}}\right), \tag{19}$$

where $Z = \frac{1}{r_0}\max\{r_{\mathrm{m}}, r_{\mathrm{s}}\}$ *with* $F_Z(z)$ *being the cdf of Z, and* r_{s} *is defined in Lemma 1. Thus, a upper bound on the critical arrival rate* $\xi_{\mathrm{c},\varepsilon}$ *is* $\xi_{\mathrm{c},\varepsilon}^{\mathrm{n1}}$, *i.e.,* $\xi_{\mathrm{c},\varepsilon} \leq \xi_{\mathrm{c},\varepsilon}^{\mathrm{n1}}$.

Proof: See Appendix C. □

The necessary condition given by Theorem 2 is not in closed form, and thus the necessary condition needs to be obtained through numerical evaluation. In the following, we derive a closed-form necessary condition by considering the further simplified system with $\varphi = \psi = -\pi$ (see Fig. 3). For a given r_m if the transmitter x_0 in the simplified system is unstable for $\varphi = \psi = -\pi$, it will also be unstable for other values of φ and ψ. This is because when $\varphi = \psi = -\pi$, the interference between the two pairs of transceivers is the smallest among all φ and ψ. The following lemma gives the sufficient and necessary condition for the queue at the transmitter x_0 to be stable when $\varphi = \psi = -\pi$ with given r_m.

Lemma 2: For the simplified system with $\varphi = \psi = -\pi$ and given r_{m} (see Fig. 3), the sufficient and necessary condition for the queue at the transmitter x_0 to be stable is

$$\xi \leq \xi_0 \frac{(1+(1-p)\theta_{\mathrm{s}})(1+\theta_{\mathrm{m}})}{p(\theta_{\mathrm{m}}-\theta_{\mathrm{s}})+(1+\theta_{\mathrm{s}})(1+\theta_{\mathrm{m}})}, \tag{20}$$

where $\theta_{\mathrm{s}} = \theta r_0^{\alpha} r_{\mathrm{s}}^{-\alpha} = \theta r_0^{\alpha}\ (r_{\mathrm{m}} + 2r_0)^{-\alpha}$ *and* $\theta_{\mathrm{m}} = \theta r_0^{\alpha} r_{\mathrm{m}}^{-\alpha}$.

Proof: This lemma is a special case of Lemma 1 obtained by setting $\varphi = \psi = -\pi$. □

In Lemma 2, the case where $\varphi = \psi = -\pi$ is considered. For any other φ and ψ with given r_{m}, (20) gives a necessary condition for the queue at the transmitter x_0 to be stable in the simplified system. Since r_{m} is a random variable and its probability distribution is given by (17), (20) gives a necessary condition for the queue at the transmitter x_0 to be stable in the simplified system. The simplified system only considers the interference from the nearest transmitter; thus (20) will also be a necessary condition for ε-stability of the original system. By modifying the proof of Theorem 2 with $r_{\mathrm{s}} = r_{\mathrm{m}} + 2r_0$, we obtain the following corollary.

Corollary 2: Given a slotted random access system with the transmitters distributed as a PPP and with Bernoulli *packet arrivals, a closed-form type I necessary condition for ε-stability is*

$$\xi \le \tilde{\xi}^{\mathrm{n1}}_{\mathrm{c},\varepsilon} \triangleq \xi_0 \left(1+\left(\frac{p\theta r_0^\alpha}{\left(\sqrt{-\frac{\ln(1-\varepsilon)}{\pi\lambda}}+2r_0\right)^\alpha+\theta r_0^\alpha}\right)^2\right)^{-1}. \tag{21}$$

Thus, a closed-form upper bound on the critical arrival rate $\xi_{\mathrm{c},\varepsilon}$ is $\tilde{\xi}^{\mathrm{n1}}_{\mathrm{c},\varepsilon}$, i.e., $\xi_{\mathrm{c},\varepsilon} \le \tilde{\xi}^{\mathrm{n1}}_{\mathrm{c},\varepsilon}$.

Proof: See Appendix D. □

B. Type II Necessary Conditions

In the following, we derive the type II necessary conditions. We consider a modified favorable system, in which the packets in the interfering transmitters that are not scheduled by random access or whose transmission failed will be dropped instead of being retransmitted; thus, an interfering transmitter is active with probability $p\xi$, decoupled from the status of other transmitters.

Theorem 3: Given a slotted random access system with the transmitters distributed as a PPP and with Bernoulli packet arrivals, a type II necessary condition for the system to be ε-stable is

$$\xi \le \xi^{\mathrm{n2}}_{\mathrm{c},\varepsilon} \triangleq \sup\Bigg\{\xi \in \mathbb{R}^+ : \frac{1}{2}-\frac{1}{\pi} \int_0^\infty \frac{1}{\omega}\mathrm{Im}\left\{\left(\frac{\xi_0}{\xi}\right)^{j\omega} e^{-j\omega\xi C_\delta {}_2F_1(1-j\omega,1-\delta;2;\xi p)}\right\}\mathrm{d}\omega \le \varepsilon\Bigg\}. \tag{22}$$

Thus, a upper bound on the critical arrival rate $\xi_{\mathrm{c},\varepsilon}$ is $\xi^{\mathrm{n2}}_{\mathrm{c},\varepsilon}$, i.e., $\xi_{\mathrm{c},\varepsilon} \le \xi^{\mathrm{n2}}_{\mathrm{c},\varepsilon}$.

Proof: See Appendix E. □

Remark 3: If $\lambda \to 0$ or $\theta \to 0$, the necessary condition in Theorem 3 coincides with the sufficient condition in Theorem 1, indicating that the two conditions are tight for small λ and θ.

The following corollary simplifies the type II necessary condition using the Markov inequality.

Corollary 3: Given a slotted random access system with the transmitters distributed as a PPP and with Bernoulli packet arrivals, a type II necessary condition for the system to be ε-stable is

$$\xi \le \xi_0(1-\varepsilon)^{-\frac{1}{t}} \exp\left(-\xi C_\delta {}_2F_1(1-t,\,1-\delta;\,2;\,\xi\, p)\right), \tag{23}$$

for all $t > 0$. For $t = 1$, we obtain a closed-form type II necessary condition as

$$\xi \le \tilde{\xi}^{\mathrm{n2}}_{\mathrm{c},\varepsilon} \triangleq \frac{1}{C_\delta}\mathcal{W}\left(\frac{C_\delta\xi_0}{1-\varepsilon}\right), \tag{24}$$

where $\mathcal{W}(z)$ is the main branch of Lambert W function. Thus, a closed-form upper bound on the critical arrival rate $\xi_{\mathrm{c},\varepsilon}$ is $\tilde{\xi}^{\mathrm{n2}}_{\mathrm{c},\varepsilon}$, i.e., $\xi_{\mathrm{c},\varepsilon} \le \tilde{\xi}^{\mathrm{n2}}_{\mathrm{c},\varepsilon}$.

Proof: See Appendix F. □

When deriving the type I necessary condition, we only considered the effect of the nearest interferer and ignored all other interferers, while in the derivation of the type II necessary condition, we considered all interferers but ignored the retransmission mechanism of the interferers. Whether the type I or the type II necessary condition should be used depends on whether the nearest interferer or the retransmission mechanism of the interferers takes the leading position in affecting the transmission. For example, when the SINR threshold θ is small and the access probability p is large, the packets will be highly likely scheduled and transmitted successfully, and no retransmission happens. Therefore, the effect of the retransmission mechanism can be neglected, and the type II necessary condition is better than the type I necessary condition.

VI. Discussion of Results

While the conditions given by the theorems are not in closed form, the corollaries give closed-form results. In order to gain insight from the results, we discuss the asymptotic behaviors and compare the sufficient and necessary conditions through numerical evaluations.

A. Asymptotic Behaviors

1) p Approaching 0: From Corollary 1, as $p \to 0$, the optimal n is $n_{\max} = \infty$. Thus, we have

$$\lim_{p\to 0} \tilde{\xi}^{\mathrm{s}}_{\mathrm{c},\varepsilon} = \xi_0. \tag{25}$$

From Corollary 2, we get

$$\lim_{p\to 0} \tilde{\xi}^{\mathrm{n1}}_{\mathrm{c},\varepsilon} = \xi_0. \tag{26}$$

From Corollary 3 and by noticing that $\lim_{z\to 0}\mathcal{W}(z)/z = 1$, we get

$$\lim_{p\to 0} \tilde{\xi}^{\mathrm{n2}}_{\mathrm{c},\varepsilon} = \frac{\xi_0}{1-\varepsilon}. \tag{27}$$

2) ε Approaching 0: Corollary 1 shows that $\tilde{\xi}^{\mathrm{s}}_{\mathrm{c},\varepsilon}$ approaches zero exponentially with attenuation factor $\frac{1}{n_{\max}}$ as $\varepsilon \to 0$. From Corollary 2, we get the asymptotic result for $\tilde{\xi}^{\mathrm{n1}}_{\mathrm{c},\varepsilon}$ as $\varepsilon \to 0$ as

$$\begin{aligned}\tilde{\xi}^{\mathrm{n1}}_{\mathrm{c},\varepsilon} &= \xi_0 \left(1+\left(\frac{p\theta r_0^\alpha}{\left(\sqrt{-\frac{\ln(1-\varepsilon)}{\pi\lambda}}+2r_0\right)^\alpha+\theta r_0^\alpha}\right)^2\right)^{-1} \\ &= \frac{(2^\alpha+\theta)^2}{p^2\theta^2+(2^\alpha+\theta)^2}\xi_0 + O\left(-\frac{\ln(1-\varepsilon)}{\pi\lambda}\right) \\ &\quad + \frac{\alpha 2^\alpha p^2\theta^2(2^\alpha+\theta)}{((2^\alpha+\theta)^2+p^2\theta^2)^2 r_0}\xi_0\sqrt{-\frac{\ln(1-\varepsilon)}{\pi\lambda}} \\ &= \frac{(2^\alpha+\theta)^2}{p^2\theta^2+(2^\alpha+\theta)^2}\xi_0 \\ &\quad + \frac{\alpha 2^\alpha p^2\theta^2(2^\alpha+\theta)}{((2^\alpha+\theta)^2+p^2\theta^2)^2 r_0\sqrt{\pi\lambda}}\xi_0\varepsilon^{\frac{1}{2}} + O(\varepsilon). \end{aligned} \tag{28}$$

(28) shows that $\tilde{\xi}^{\mathrm{n1}}_{\mathrm{c},\varepsilon}$ approaches $\frac{(2^\alpha+\theta)^2}{p^2\theta^2+(2^\alpha+\theta)^2}\xi_0$ with residual $\Theta(\varepsilon^{\frac{1}{2}})$.

From Corollary 3, letting $z_0 = \xi_0 C_\delta$, we get the asymptotic results for $\widetilde{\xi}^{\mathrm{n2}}_{\mathrm{c},\varepsilon}$ as $\varepsilon \to 0$ as

$$\widetilde{\xi}^{\mathrm{n2}}_{\mathrm{c},\varepsilon} \overset{(a)}{=} \frac{\mathcal{W}(z_0)}{C_\delta} + \frac{\mathcal{W}(z_0)\, z_0}{C_\delta z_0 (1+\mathcal{W}(z_0))}\frac{\varepsilon}{1-\varepsilon} + O\left(\frac{\varepsilon}{1-\varepsilon}\right) = \frac{\mathcal{W}(z_0)}{C_\delta} + \frac{\mathcal{W}(z_0)}{C_\delta(1+\mathcal{W}(z_0))}\varepsilon + O(\varepsilon^2) \quad (29)$$

where (a) follows from the Taylor expansion of $\mathcal{W}(z)$ at z_0, i.e. $\mathcal{W}(z) = \mathcal{W}(z_0) + \frac{\mathcal{W}(z_0)(z-z_0)}{z_0(1+\mathcal{W}(z_0))} + O((z-z_0)^2)$ as $z \to z_0$. (29) shows that $\widetilde{\xi}^{\mathrm{n2}}_{\mathrm{c},\varepsilon}$ approaches $\frac{\mathcal{W}(z_0)}{C_\delta}$ with residual $\Theta(\varepsilon)$.

3) λ Approaching 0: From Corollary 1, as $\lambda \to 0$, the optimal n to maximize $\eta(n)$ is $n_{\max} = \infty$. The asymptotic result for $\widetilde{\xi}^{\mathrm{s}}_{\mathrm{c},\varepsilon}$ as $\lambda \to 0$ is

$$\widetilde{\xi}^{\mathrm{s}}_{\mathrm{c},\varepsilon} = \xi_0\Bigg(1 - \pi\lambda\delta(1-p)^\delta\theta^\delta r_0^2 \lim_{n\to\infty}\sum_{i=1}^{n}((1-p)^{-i}-1) \times \frac{\Gamma(i-\delta)\Gamma(n-i+\delta)}{\Gamma(i+1)\Gamma(n-i+1)} + O(\lambda^2)\Bigg), \quad (30)$$

which reveals that $\widetilde{\xi}^{\mathrm{s}}_{\mathrm{c},\varepsilon}$ approaches ξ_0 with a factor of $1-\Theta(\lambda)$.

From Corollary 2, we get the asymptotic result for $\widetilde{\xi}^{\mathrm{n1}}_{\mathrm{c},\varepsilon}$ as $\lambda \to 0$ as

$$\widetilde{\xi}^{\mathrm{n1}}_{\mathrm{c},\varepsilon} = \xi_0\left(1 - \frac{p^2\theta^2 r_0^{2\alpha}\pi^\alpha}{(-\ln(1-\varepsilon))^\alpha}\lambda^\alpha + O(\lambda^{2\alpha})\right), \quad (31)$$

indicating that $\widetilde{\xi}^{\mathrm{n1}}_{\mathrm{c},\varepsilon}$ approaches ξ_0 with a factor of $1-\Theta(\lambda^\alpha)$.

From Corollary 3 and by noticing that $\mathcal{W}(z) = z - O(z^2)$ as $z \to 0$, we get the asymptotic results for $\widetilde{\xi}^{\mathrm{n2}}_{\mathrm{c},\varepsilon}$ as $\lambda \to 0$ as

$$\widetilde{\xi}^{\mathrm{n2}}_{\mathrm{c},\varepsilon} = \frac{1}{1-\varepsilon}\xi_0 - \frac{1}{(1-\varepsilon)^2}p\lambda\pi r_0^2\theta^\delta\Gamma(1+\delta)\Gamma(1-\delta)(\xi_0)^2 + O(\lambda^2), \quad (32)$$

which reveals that $\widetilde{\xi}^{\mathrm{n2}}_{\mathrm{c},\varepsilon}$ approaches $\frac{1}{1-\varepsilon}\xi_0$ with residual $\Theta(\lambda)$.

4) θ Approaching 0: When fixing the duration of each time slot and varying θ, we multiply $\widetilde{\xi}^{\mathrm{s}}_{\mathrm{c},\varepsilon}$ with the factor $\log_2(1+\theta)$ since the size of each packet is changed. The factor $\log_2(1+\theta)$ guarantees that when varying θ, arrival rates with different packet sizes are compared fairly. From Corollary 1, as $\theta \to 0$, the optimal n to maximize $\eta(n)$ is $n_{\max} = \infty$. We get the asymptotic results for $\widetilde{\xi}^{\mathrm{s}}_{\mathrm{c},\varepsilon}\log_2(1+\theta)$ as $\theta \to 0$ as

$$\widetilde{\xi}^{\mathrm{s}}_{\mathrm{c},\varepsilon}\log_2(1+\theta) = \frac{p}{\ln 2}\theta + O(\theta^2). \quad (33)$$

From Corollary 2, we get the asymptotic results for $\widetilde{\xi}^{\mathrm{n1}}_{\mathrm{c},\varepsilon}\log_2(1+\theta)$ as $\theta \to 0$ as

$$\widetilde{\xi}^{\mathrm{n1}}_{\mathrm{c},\varepsilon}\log_2(1+\theta) = \frac{p}{\ln 2}\theta + O(\theta^2). \quad (34)$$

From Corollary 3 and by noticing that $\lim_{z\to 0}\mathcal{W}(z)/z = 1$, we get the asymptotic results for $\widetilde{\xi}^{\mathrm{n2}}_{\mathrm{c},\varepsilon}\log_2(1+\theta)$ as $\theta \to 0$ as

$$\widetilde{\xi}^{\mathrm{n2}}_{\mathrm{c},\varepsilon}\log_2(1+\theta) = \frac{p\theta}{(1-\varepsilon)\ln 2} + O(\theta^2). \quad (35)$$

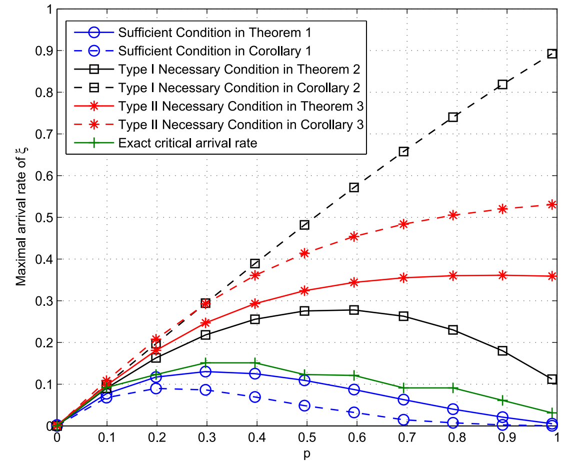

Fig. 4. Comparison of sufficient conditions and necessary conditions as functions of p. The parameters are set as $\varepsilon = 0.1$, $\theta = 15$dB, $r_0 = 1$, $W = 0$, $\alpha = 4$ and $\lambda = 0.05$.

Therefore, $\widetilde{\xi}^{\mathrm{s}}_{\mathrm{c},\varepsilon}\log_2(1+\theta)$ and $\widetilde{\xi}^{\mathrm{n1}}_{\mathrm{c},\varepsilon}\log_2(1+\theta)$ approach 0 linearly with the same slope coefficient $\frac{p}{\ln 2}$, while $\widetilde{\xi}^{\mathrm{n2}}_{\mathrm{c},\varepsilon}\log_2(1+\theta)$ approaches 0 linearly with the slope coefficient $\frac{p}{(1-\varepsilon)\ln 2}$.

B. Comparison of Sufficient and Necessary Conditions

In this subsection, we numerically compare the sufficient conditions and the necessary conditions derived in the previous sections. By the Monte Carlo simulation, we also plot the curves for the exact critical arrival rates, which are obtained by gradually increase the arrival rate until the condition for ε-stability of the network is not satisfied.

Fig. 4 shows the maximal arrival rates per the sufficient conditions and necessary conditions as functions of p. As $p \to 0$, all curves converge to 0, as explained in subsubsection VI-A1. As p increases, the curves for the non-closed-form sufficient condition (solid line with circle marks) and for the type I non-closed-form necessary condition (solid line with square marks) first increase then decrease, because the success probability is limited by the small access probability for small p and by the large interference for large p.

Fig. 5 plots the maximal arrival rates per the sufficient conditions and necessary conditions as functions of ε. As $\varepsilon \to 0$, the curves for the sufficient conditions and the non-closed-form type II necessary condition approach 0, and other curves approach different constant values. Fig. 5 reveals that the curves do not depend strongly on ε. Since the gap between the curves for the sufficient conditions and that for the necessary conditions is not large, it can be inferred that the critical arrival rate for actual ε-stability region does not change much either when increasing ε. This observation indicates that a slight change in the arrival rate ξ may greatly affect the fraction of unstable queues in the network.

Fig. 6 plots the maximal arrival rates per the sufficient conditions and necessary conditions as functions of λ. We observe that all curves except the one for the closed-form type II necessary condition converge to the same value. This is because as $\lambda \to 0$, the interference is negligible, and only

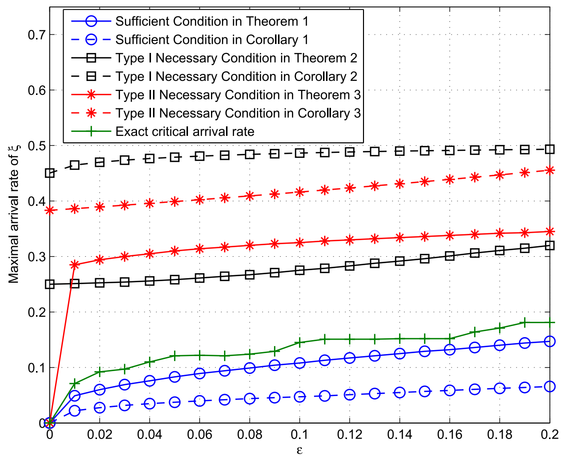

Fig. 5. Comparison of sufficient conditions and necessary conditions as functions of ε. The parameters are set as $p = 0.5$, $\theta = 15$dB, $r_0 = 1$, $W = 0$, $\alpha = 4$ and $\lambda = 0.05$.

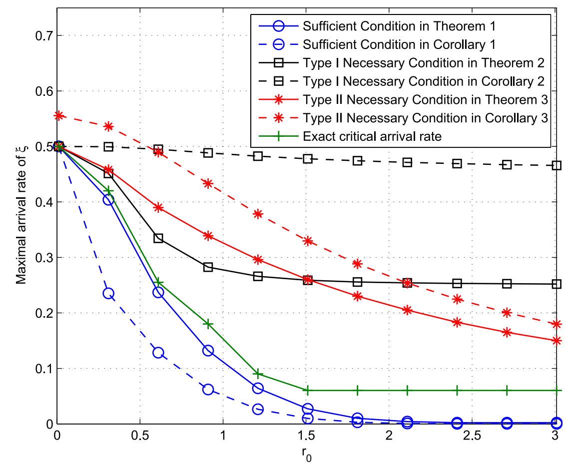

Fig. 7. Comparison of sufficient conditions and necessary conditions as functions of r_0. The parameters are set as $p = 0.5$, $\varepsilon = 0.1$, $\theta = 15$dB, $\lambda = 0.05$, $W = 0$ and $\alpha = 4$.

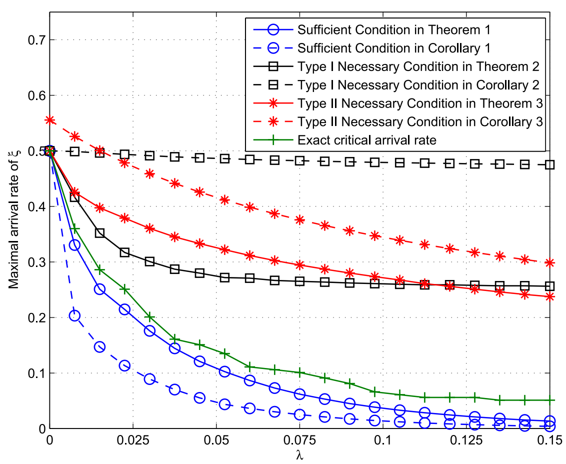

Fig. 6. Comparison of sufficient conditions and necessary conditions as functions of λ. The parameters are set as $p = 0.5$, $\varepsilon = 0.1$, $\theta = 15$dB, $r_0 = 1$, $W = 0$ and $\alpha = 4$.

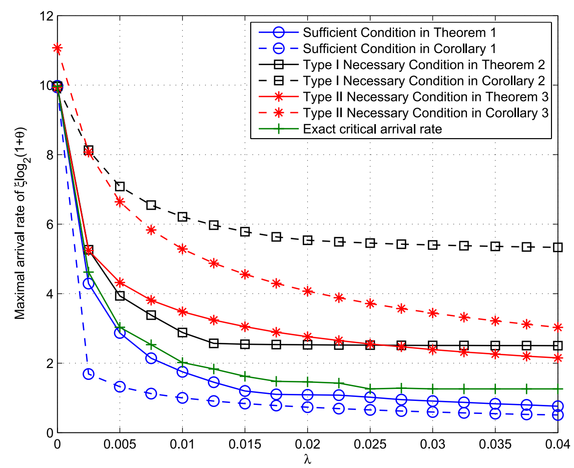

Fig. 8. Comparison of sufficient condition and necessary condition as a function of λ with optimal pair of (p, θ). The parameters are set as $\varepsilon = 0.1$, $r_0 = 1$, $W = 0$ and $\alpha = 4$.

the noise affects the transmission for the dominant system and the simplified system; however, the curve for the closed-form type II necessary condition does not converge to the same value as $\lambda \to 0$ because of the use of the Markov inequality in the derivation.

Fig. 7 plots the maximal arrival rates per the sufficient conditions and necessary conditions as functions of the distance between the transmitter and the receiver r_0. We observe that the type I necessary condition is better than the type II necessary condition for small r_0 and worse for large r_0. It can be interpreted as that when the distance between the transmitter and receiver r_0 is small, the power of the useful signal is large, and the interference from most of the transmitters is negligible compared to the useful signal. Therefore, considering only the nearest interferer, which is the case of the type I necessary condition, will result in a better approximation when r_0 is small. On the other hand, when r_0 is large, the effect of more interfering transmitters cannot be negligible; thus, the type II necessary condition will become better.

For the case where p and θ can be optimized, i.e., the transmit probability p and the SINR threshold θ are designable parameters that can be chosen to maximize the maximal arrival rate times $\log_2(1+\theta)$. To obtain realistic values, we choose θ from $[-20, 30]$ dB; then, Fig. 8 plots the maximal arrival rates times $\log_2(1+\theta)$ in terms of sufficient conditions and necessary conditions as functions of λ when the optimal p and θ are chosen.

As $\varepsilon \to 0$, the type II necessary condition is better than the type I necessary condition since the arrival rate can be positive to make the network strictly stable ($\varepsilon = 0$) when only the nearest interferer is considered, which is not realistic in the original system. As $p \to 0$, a packet is dropped with high probability, and as $\lambda \to 0$, the interference caused by the interferers except the nearest one is negligible; thus, in these cases, the type I necessary condition is better. As $\theta \to 0$ and $p \to 1$, the dropping of packets rarely happens, and if r_0 is larger than the mean distance to the nearest interferer $1/(2\sqrt{\lambda})$, other interferers cannot be ignored; thus in these cases, the type II necessary condition will be better. We summarize the results in Table I, which lists some situations where it is preferable to use one of the two types of necessary conditions.

TABLE I
SOME SITUATIONS TO USE TYPE I OR TYPE II NECESSARY CONDITIONS

Case	Type	Case	Type
$\varepsilon \to 0$	Type II	$p \to 0$	Type I
$\lambda \to 0$	Type I	$\theta \to 0$ and $p \to 1$	Type II
$\lambda > 1/(4r_0^2)$	Type II		

VII. CONCLUSIONS

In this paper, we investigated the stable packet arrival rate region of a discrete-time slotted random access network with transmitters and receivers distributed as a static Poisson bipolar process. We introduced the notion of ε-stability, and obtained sufficient conditions and two types of necessary conditions for ε-stability. The asymptotic behaviors show that the obtained sufficient conditions and necessary conditions converge to the exact condition for ε-stability when some extreme system parameters are chosen. The numerical results reveal that the gap between the sufficient conditions and the necessary conditions is small when the access probability, the density of transmitters or the SINR threshold is small. The results also reveal that a slight change in the arrival rate may greatly affect the fraction of unstable queues in the network. Moreover, since we have two kinds of necessary conditions, we provide some guidance on whether to use the type I or type II necessary conditions for a specific network scenario.

APPENDIX A
PROOF OF THEOREM 1

The success probability for the typical transmission conditioned on Φ in the dominant system is denoted as $\mathbb{P}^{x_0}(\mathcal{C}_\Phi) = p\mathbb{P}^{x_0}(\text{SINR} > \theta \mid \Phi)$, which is evaluated as

$$\begin{aligned}
\mathbb{P}^{x_0}(\mathcal{C}_\Phi) &= p\mathbb{P}^{x_0}\left(h_{k,x_0} r_0^{-\alpha} > \theta\,(W + I_k) \mid \Phi\right)\\
&\overset{(a)}{=} p\mathbb{E}^{x_0}\left[\exp\left(-\theta r_0^\alpha\,(W + I_k)\right) \mid \Phi\right]\\
&= p\mathbb{E}^{x_0}\Big[\exp\Big(-\theta r_0^\alpha W\\
&\quad - \sum_{x\in\Phi\backslash\{x_0\}} \theta r_0^\alpha h_{k,x}|x|^{-\alpha}\mathbf{1}(x\in\Phi_k)\Big) \mid \Phi\Big]\\
&= \xi_0 \prod_{x\in\Phi\backslash\{x_0\}} \Big(p\mathbb{E}^{x_0}\Big[\exp\big(-\theta r_0^\alpha h_{k,x}|x|^{-\alpha}\big) \mid \Phi\Big] + 1 - p\Big)\\
&\overset{(b)}{=} \xi_0 \prod_{x\in\Phi\backslash\{x_0\}} \Big(\frac{p}{1+\theta r_0^\alpha |x|^{-\alpha}} + 1 - p\Big).
\end{aligned} \tag{36}$$

where (a) and (b) follow because the fading coefficients $\{h_{k,x}\}$ are i.i.d. exponential distributed random variables with unit mean. The moment generating function of $Y \overset{\Delta}{=} \ln(\mathbb{P}^{x_0}(\mathcal{C}_\Phi))$ is

$$\begin{aligned}
M_Y(s) &= \mathbb{E}\left[e^{s\ln(\mathbb{P}^{x_0}(\mathcal{C}_\Phi))}\right]\\
&= (\xi_0)^s \mathbb{E}\Big[\prod_{x\in\Phi\backslash\{x_0\}} \Big(\frac{p}{1+\theta r_0^\alpha|x|^{-\alpha}} + 1 - p\Big)^s\Big]\\
&= (\xi_0)^s \exp\Big(-2\pi\lambda \int_0^\infty \Big(1 - \Big(\frac{p}{1+\theta r_0^\alpha r^{-\alpha}} + 1 - p\Big)^s\Big) r\,\mathrm{d}r\Big)\\
&= (\xi_0)^s \exp\Big(-sC_\delta\, {}_2F_1(1-s,\, 1-\delta;\, 2;\, p)\Big).
\end{aligned} \tag{37}$$

The cdf of Y, denoted by $F_Y(y) = \mathbb{P}(Y \le y)$, follows from the Gil-Pelaez Theorem [32] as

$$F_Y(y) = \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\{e^{-j\omega y} M_Y(j\omega)\}}{\omega}\mathrm{d}\omega. \tag{38}$$

The probability that the transmitter x_0 in the dominant system is unstable is given by the cdf of $\mathbb{P}^{x_0}(\mathcal{C}_\Phi)$, which is

$$\begin{aligned}
\mathbb{P}^{x_0}\left\{\mathbb{P}^{x_0}(\mathcal{C}_\Phi) \le \xi\right\} &= \mathbb{P}^{x_0}\{Y \le \ln\xi\}\\
&= \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\{e^{-j\omega\ln\xi} M_Y(j\omega)\}}{\omega}\mathrm{d}\omega.
\end{aligned} \tag{39}$$

The condition for the queue at the typical transmitter in the dominant system to be stable is $\mathbb{P}^{x_0}\{\mathbb{P}^{x_0}(\mathcal{C}_\Phi) \le \xi\} \le \varepsilon$. By combining (37) and (39), we obtain

$$\begin{aligned}
&\frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{1}{\omega}\mathrm{Im}\Big\{(\xi_0)^{j\omega}\exp\big(-j\omega\ln\xi\\
&\qquad - j\omega C_\delta\, {}_2F_1(1-j\omega,\, 1-\delta;\, 2;\, p)\big)\Big\}\mathrm{d}\omega \le \varepsilon.
\end{aligned} \tag{40}$$

Therefore, we get the results in the theorem.

APPENDIX B
PROOF OF LEMMA 1

In the dominant simplified system, the transmitter x_0 is active with probability p. The probability that the nearest interfering transmitter x_1 is scheduled and also successful is

$$\begin{aligned}
p_1 &= p^2\mathbb{P}\left\{\frac{h_1 r_0^{-\alpha}}{h_2 r_\mathrm{s}^{-\alpha} + W} > \theta\right\} + p(1-p)\mathbb{P}\left\{\frac{h_1 r_0^{-\alpha}}{W} > \theta\right\}\\
&\overset{(a)}{=} \xi_0\left(\frac{p}{1+\theta r_0^\alpha r_\mathrm{s}^{-\alpha}} + 1 - p\right).
\end{aligned} \tag{41}$$

where h_1 is the fading coefficient between the transmitter and the receiver of the nearest pair of transceiver, and h_2 is the fading coefficient between the transmitter x_0 and the receiver y_1. (a) follows because h_1 and h_2 are both exponentially distributed. In the following, we divide the proof into two cases, i.e., $\xi \ge p_1$ and $\xi < p_1$.

1) The Case Where $\xi \ge p_1$: When $\xi \ge p_1$, the queue at the nearest interfering transmitter x_1 is unstable and will never be empty, thus x_1 will cause interference to the typical transmission with probability p. Therefore, when $\xi \ge p_1$ the probability that the transmitter x_0 is scheduled and successful is

$$\begin{aligned}
p_0 &= p^2\mathbb{P}\left\{\frac{h_3 r_0^{-\alpha}}{h_4 r_\mathrm{m}^{-\alpha} + W} > \theta\right\} + p(1-p)\mathbb{P}\left\{\frac{h_3 r_0^{-\alpha}}{W} > \theta\right\}\\
&= \xi_0\left(\frac{p}{1+\theta r_0^\alpha r_\mathrm{m}^{-\alpha}} + 1 - p\right).
\end{aligned} \tag{42}$$

where h_3 is the fading coefficient between the transmitter x_0 and the receiver y_0, and h_4 is the fading coefficient between the nearest interfering transmitter x_1 and the receiver y_0.

If $r_s > r_m$, by comparing (41) with (42), we have $\xi > p_1 > p_0$, which implies that the queue at the transmitter x_0 is unstable if $\xi \geq p_1$. This can be explained intuitively by the concept of "stability rank" [9], which states that if a queue is unstable, the queues with higher rank than the said queue are unstable as well. The interference from the transmitter x_0 to the receiver y_1 is less than that from the nearest interfering transmitter x_1 to the receiver y_0 when $r_s > r_m$, which means that the queue at x_0 has higher rank than the queue at x_1; Thus, when the queue at the nearest interferer x_1 is unstable, the queue at the transmitter x_0 is also unstable.

If $r_s \leq r_m$, the queue at x_1 has higher rank than the queue at x_0. By comparing (41) with (42), we have $p_0 \geq p_1$, which implies that the queue at the transmitter x_0 is stable for $p_0 \geq \xi \geq p_1$ and unstable for $\xi > p_0$.

2) The Case Where $\xi < p_1$: When $\xi < p_1$, the queue of the nearest interfering transmitter x_1 is empty with probability $1 - \xi/p_1$ and is nonempty with probability ξ/p_1. Therefore, when $\xi < p_1$ the probability that the transmitter x_0 is scheduled by random access and successful is

$$\begin{aligned} p_0' &= p^2 \frac{\xi}{p_1} \mathbb{P}\left\{ \frac{h_3 r_0^{-\alpha}}{h_4 r_m^{-\alpha} + W} > \theta \right\} \\ &\quad + \left(p(1-p)\frac{\xi}{p_1} + p\left(1 - \frac{\xi}{p_1}\right)\right) \mathbb{P}\left\{ \frac{h_3 r_0^{-\alpha}}{W} > \theta \right\} \\ &= \xi_0 \left(\frac{p\xi}{p_1} \frac{1}{1+\theta r_0^{\alpha} r_m^{-\alpha}} + 1 - \frac{p\xi}{p_1} \right). \end{aligned} \tag{43}$$

To make the queue at the transmitter x_0 stable, the arrival rate should satisfy $\xi \leq p_0'$, i.e.,

$$\begin{aligned} \xi &\leq \frac{p p_1}{p_1 \exp\left(W\theta r_0^{\alpha}\right) + p^2 - p^2 \frac{1}{1+\theta r_0^{\alpha} r_m^{-\alpha}}} \\ &= p_1 \left(\frac{p}{1+\theta r_0^{\alpha} r_s^{-\alpha}} - \frac{p}{1+\theta r_0^{\alpha} r_m^{-\alpha}} + 1 \right)^{-1} \\ &= \xi_0 \left(\frac{p}{1+\theta r_0^{\alpha} r_s^{-\alpha}} + 1 - p \right) \\ &\quad \times \left(\frac{p}{1+\theta r_0^{\alpha} r_s^{-\alpha}} - \frac{p}{1+\theta r_0^{\alpha} r_m^{-\alpha}} + 1 \right)^{-1}. \end{aligned} \tag{44}$$

If $r_s > r_m$, it can be verified that the right side of the above inequality is less than p_1. Therefore, if $\xi < p_1$, the queue at the transmitter x_0 in the dominant simplified system will be stable only when the inequality (44) is satisfied.

If $r_s \leq r_m$, the right side of the above inequality is larger than p_1. Therefore, for the case $\xi < p_1$, the queue at the transmitter x_0 in the dominant simplified system will be stable.

Combining the cases $\xi \geq p_1$ and $\xi < p_1$, the queue at x_0 in the dominant simplified system is stable if and only if

$$\xi \leq \begin{cases} \xi_0 \left(\frac{p}{1+\theta r_0^{\alpha} r_s^{-\alpha}} + 1 - p \right) \\ \quad \cdot \left(\frac{p}{1+\theta r_0^{\alpha} r_s^{-\alpha}} - \frac{p}{1+\theta r_0^{\alpha} r_m^{-\alpha}} + 1 \right)^{-1} & \text{if } r_s > r_m \\ \xi_0 \left(\frac{p}{1+\theta r_0^{\alpha} r_m^{-\alpha}} + 1 - p \right) & \text{if } r_s \leq r_m \end{cases} \tag{45}$$

Thus, (45) also gives the sufficient and necessary condition for the queue at the transmitter x_0 to be stable in the original simplified system.

Appendix C
Proof of Theorem 2

According to Lemma 1, if $r_s > r_m$, from (45) we have

$$\begin{aligned} \xi &\leq \xi_0 \left(\frac{p}{1+\theta r_0^{\alpha} r_s^{-\alpha}} + 1 - p \right) \\ &\quad \times \left(\frac{p}{1+\theta r_0^{\alpha} r_s^{-\alpha}} - \frac{p}{1+\theta r_0^{\alpha} r_m^{-\alpha}} + 1 \right)^{-1} \\ &\leq \xi_0 \left(\frac{p}{1+\theta r_0^{\alpha} r_s^{-\alpha}} + 1 - p \right). \end{aligned} \tag{46}$$

Since Lemma 1 gives a sufficient and necessary condition for the transmitter x_0 to be stable in the simplified system when φ, ψ, r_m are given, comparing (45) and (46), we obtain a necessary condition as

$$\xi \leq \xi_0 \left(\frac{p}{1+\theta r_0^{\alpha} \left(\max\{r_m, r_s\}\right)^{-\alpha}} + 1 - p \right). \tag{47}$$

According to (3) and Lemma 1, when φ, ψ, r_m are random variables, a necessary condition for the simplified system to be stable is

$$\begin{aligned} \varepsilon &\geq \mathbb{P}\left\{ \xi \geq \xi_0 \left(\frac{p}{1+\theta r_0^{\alpha} \left(\max\{r_m, r_s\}\right)^{-\alpha}} + 1 - p \right) \right\} \\ &= \mathbb{P}\left\{ \frac{1}{r_0} \max\{r_m, r_s\} \leq \left(\theta \frac{\xi + p\xi_0 - \xi_0}{\xi_0 - \xi} \right)^{1/\alpha} \right\}. \end{aligned} \tag{48}$$

Let $Z = \frac{1}{r_0} \max\{r_m, r_s\}$, and denote the cdf of Z as $F_Z(z)$. (48) can be written as

$$\varepsilon \geq F_Z \left(\frac{1}{r_0} \max\{r_m, r_s\} \leq \left(\theta \frac{\xi + p\xi_0 - \xi_0}{\xi_0 - \xi} \right)^{1/\alpha} \right), \tag{49}$$

which is equivalent to

$$\xi \leq \xi_0 \left(1 - \frac{\theta p}{\theta + \left(F_Z^{-1}(\varepsilon) \right)^{\alpha}} \right). \tag{50}$$

Therefore, we obtain the necessary condition in Theorem 2.

APPENDIX D
PROOF OF COROLLARY 2

According to (3) and Lemma 2, r_{m} is a random variable in the original system. Thus, a necessary condition for the original system to be ε-stable is

$$\varepsilon \geq \mathbb{P}\left\{\xi \geq \frac{\left(r_{\mathrm{m}}^{\alpha}+\theta r_0^{\alpha}\right)\left((r_{\mathrm{m}}+2r_0)^{\alpha}+(1-p)\theta r_0^{\alpha}\right)\xi_0}{\left(r_{\mathrm{m}}^{\alpha}+(1+p)\theta r_0^{\alpha}\right)\left((r_{\mathrm{m}}+2r_0)^{\alpha}+(1-p)\theta r_0^{\alpha}\right)+p^2\theta^2 r_0^{2\alpha}}\right\}. \tag{51}$$

Since $f(x) = \frac{x}{1+x}$ is an increasing function, we obtain a necessary condition for the original system to be ε-stable as

$$\varepsilon > \mathbb{P}\left\{\xi \geq \frac{\left((r_{\mathrm{m}}+2r_0)^{\alpha}+\theta r_0^{\alpha}\right)^2 \xi_0}{\left((r_{\mathrm{m}}+2r_0)^{\alpha}+\theta r_0^{\alpha}\right)^2+p^2\theta^2 r_0^{2\alpha}}\right\} = \mathbb{P}\left\{(\xi_0-\xi)\left((r_{\mathrm{m}}+2r_0)^{\alpha}+\theta r_0^{\alpha}\right)^2 \leq \xi p^2\theta^2 r_0^{2\alpha}\right\}. \tag{52}$$

Since the inequality $\xi_0 - \xi > 0$ is satisfied from (4), we have

$$\varepsilon > \mathbb{P}\Bigg\{r_{\mathrm{m}} \leq \underbrace{\left(\sqrt{\frac{\xi p^2\theta^2 r_0^{2\alpha}}{\xi_0-\xi}}-\theta r_0^{\alpha}\right)^{1/\alpha}-2r_0}_{A}\Bigg\}. \tag{53}$$

When $A \leq 0$, the probability at the right side of the inequality is zero; thus the above inequality (53) always holds. When $A > 0$, using the probability distribution of r_{m} given by (17), we have

$$\varepsilon > 1-\exp\left(-\pi\lambda A^2\right), \tag{54}$$

and thus

$$0 < A < \sqrt{-\frac{\ln(1-\varepsilon)}{\pi\lambda}}. \tag{55}$$

Combining the cases of $A \leq 0$ and $A > 0$, we have

$$\left(\sqrt{\frac{\xi p^2\theta^2 r_0^{2\alpha}}{\xi_0-\xi}}-\theta r_0^{\alpha}\right)^{1/\alpha}-2r_0 < \sqrt{-\frac{\ln(1-\varepsilon)}{\pi\lambda}}. \tag{56}$$

Solving the above inequality, we get the result in the corollary.

APPENDIX E
PROOF OF THEOREM 3

By introducing the modified favorable system, an interfering transmitter is active with probability ξp. Similar to the derivation of (36), we get

$$\begin{aligned}\mathbb{P}^{x_0}(\mathcal{C}_\Phi) &= \xi_0 \prod_{x\in\Phi\setminus\{x_0\}}\left(\xi p\mathbb{E}^{x_0}\left[\exp\left(-\theta r_0^{\alpha}h_{k,x}|x|^{-\alpha}\right)\mid\Phi\right]+1-\xi p\right)\\ &= \xi_0 \prod_{x\in\Phi\setminus\{x_0\}}\left(\frac{\xi p}{1+\theta r_0^{\alpha}|x|^{-\alpha}}+1-\xi p\right).\end{aligned} \tag{57}$$

Letting $Y \triangleq \ln(\mathbb{P}^{x_0}(\mathcal{C}_\Phi))$, the moment generating function of Y is

$$M_Y(s) = (\xi_0)^s \exp\left(-\xi s C_{\delta 2}F_1(1-s,\,1-\delta;\,2;\,\xi p)\right). \tag{58}$$

The cdf of Y can be derived as follows by applying the Gil-Pelaez Theorem given by (38).

$$F_Y(y) = \frac{1}{2}-\frac{1}{\pi}\int_0^{\infty}\frac{\mathrm{Im}\{e^{-j\omega y}M_Y(j\omega)\}}{\omega}\mathrm{d}\omega. \tag{59}$$

The probability that the queue at the typical transmitter in the modified system is unstable is

$$\begin{aligned}&\mathbb{P}^{x_0}\left\{\mathbb{P}^{x_0}(\mathcal{C}_\Phi)\leq\xi\right\}\\ &= \mathbb{P}^{x_0}\left\{\ln\left(\mathbb{P}^{x_0}(\mathcal{C}_\Phi)\right)\leq\ln\xi\right\}\\ &= \frac{1}{2}-\frac{1}{\pi}\int_0^{\infty}\frac{\mathrm{Im}\{e^{-j\omega\ln\xi}M_Y(j\omega)\}}{\omega}\mathrm{d}\omega.\end{aligned} \tag{60}$$

The condition for the queue at the typical transmitter in the modified system to be stable is $\mathbb{P}^{x_0}\{\mathbb{P}^{x_0}(\mathcal{C}_\Phi)\leq\xi\}\leq\varepsilon$. By combining (58) and (60), we get the condition for the queue at the typical transmitter in the modified system to be stable as

$$\frac{1}{2}-\frac{1}{\pi}\int_0^{\infty}\frac{1}{\omega}\mathrm{Im}\Big\{(\xi_0)^{j\omega}\exp\big(-j\omega\ln\xi - j\omega\xi C_{\delta 2}F_1(1-j\omega,\,1-\delta;\,2;\,\xi p)\big)\Big\}\mathrm{d}\omega\leq\varepsilon. \tag{61}$$

Therefore, we get the necessary condition for the original system to be ε-stable.

APPENDIX F
PROOF OF COROLLARY 3

For all $t > 0$, by applying Markov inequality, we obtain

$$\begin{aligned}&\mathbb{P}^{x_0}\left\{\mathbb{P}^{x_0}(\mathcal{C}_\Phi)<\xi\right\}\\ &= \mathbb{P}^{x_0}\left\{(\mathbb{P}^{x_0}(\mathcal{C}_\Phi))^t<\xi^t\right\}\\ &> 1-\xi^{-t}\mathbb{E}\left[\left(\mathbb{P}^{x_0}(\mathcal{C}_\Phi)\right)^t\right]\\ &= 1-(\xi_0)^t\xi^{-t}\mathbb{E}\Bigg[\prod_{x\in\Phi\setminus\{x_0\}}\left(\frac{\xi p}{1+\theta r_0^{\alpha}|x|^{-\alpha}}+1-\xi p\right)^t\Bigg]\\ &= 1-(\xi_0)^t\xi^{-t}\exp\left(-\xi t C_{\delta 2}F_1(1-t,\,1-\delta;\,2;\,\xi p)\right).\end{aligned} \tag{62}$$

Solving the following inequality, we get a type II necessary condition given by (23):

$$1-(\xi_0)^t\xi^{-t}\exp\left(-\xi t C_{\delta 2}F_1(1-t,\,1-\delta;\,2;\,\xi p)\right)\leq\varepsilon.$$

When $t = 1$, we obtain

$$\mathbb{P}^{x_0}\left\{\mathbb{P}^{x_0}(\mathcal{C}_\Phi)<\xi\right\} > 1-\xi_0\xi^{-1}\exp(-\xi C_\delta). \tag{63}$$

Let $\mathcal{W}(z)$ be the main branch of Lambert W function, defined by $z = \mathcal{W}(z)e^{\mathcal{W}(z)}$ for any complex number z. Solving the inequality

$$1-\xi_0\xi^{-1}\exp(-\xi C_\delta)\leq\varepsilon, \tag{64}$$

we get a closed-form type II necessary condition in the corollary.

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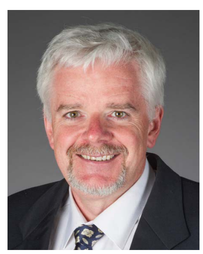

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