Secondary Users Entering the Pool: A Joint Optimization Framework for Spectrum Pooling

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***Abstract*—Spectrum pooling has been shown to have a great potential to improve the spectrum utilization, especially when primary users (PUs) and secondary users (SUs) are allowed to utilize a common spectrum pool. This paper studies the joint optimization problem for a spectrum pooling system with both PUs and SUs. We develop a novel hierarchical game theoretic model which consists of an overlapped coalition formation game model to analyze the pricing cooperation/competition strategy among PUs and a non-cooperative game model to investigate the resource competition among SUs. These two game models are interrelated in a hierarchical game structure, in which we also study the interaction between SUs and PUs. Our model does not require SUs to have information about spectrum access scheduling of PUs. Furthermore, we propose a simple distributed joint optimization algorithm that can optimize the coalition formation of PUs as well as the sub-band allocation and transmit powers of SUs. To study different fairness criteria and their effects on the payoff divisions among PUs, we derive the optimal payoff division schemes of two popular fairness criteria, namely Nash bargaining solution and Shapley value fairness.**

***Index Terms*—Cognitive radio, spectrum pooling, power control, sub-band allocation, price adjustment, spectrum sharing, game theory, Stackelberg game, coalition formation.**

I. INTRODUCTION

Spectrum pooling is a technology that allows multiple spectrum owners to merge their spectrum to form a common pool. Different from the traditional frequency-division (FD) based cognitive radio (CR) network framework (shown in Figure 1 (a)), in which each primary user (PU) can only access its exclusively licensed spectrum, a spectrum pooling system (shown in Figure 1 (b)) allows multiple PUs to coexist in the same spectrum, and hence has the potential to improve spectrum utilization and efficiency.

Most of the current works of the spectrum pooling focus on the economic incentive of PUs to trade their spectrum and assume that no unlicensed spectrum user, referred to as secondary user (SU), exists in the system [2]–[5]. However, in many existing wireless networks, simply allowing multiple PUs to access a common spectrum pool cannot provide a complete solution to the spectrum under-utilization problem. For example, in cellular networks, the operators build the infrastructure based on the worst-case scenario to support the service demand during the peak hours (e.g., the Christmas and new year's eve) and hence the resource and infrastructure remain under-utilized for most of the time. In other words, the traffic patterns of different operators in the same region can be highly correlated. For example, different operators can experience similar peak and non-peak periods in their networks. In this scenario, using spectrum pooling can improve the spectrum efficiency during the peak hours. It will, however, result in more empty spectral space in the licensed spectrum pool [6] during the non-peak hours. Therefore, the PUs in the spectrum pooling system should be more willing to sell their available spectrum to SUs.

There are several problems when SUs are allowed to access a spectrum pool. From the PUs' point of view, the spectrum pool is generally formed by combing the licensed spectrum of multiple PUs, and hence the revenue obtained from SUs should be shared by all the PUs. From the SUs' point of view, the dynamic access of the PUs in the spectrum pool makes SUs impossible to accurately estimate the detailed transmission scheduling of each PU. For example, in Figure 1 (b), the SUs cannot know the exact frequency band and transmission scheduling of every PU, and hence the transmission of each SU will have a high chance to cause interference to multiple PUs. This causes another problem because, generally speaking, different PUs have different specifications such as locations, signal-to-noise ratio (SNR) walls, capacities on tolerating the interference etc. Also, different PUs may want to sell the spectrum to SUs for different purposes. In other words, the pricing competition among PUs for SUs in the spectrum pooling system is much more complex than that of FD-based CR networks.

This motivates the work in this paper where we focus on the joint optimization of a general spectrum pooling system with the existence of both PUs and SUs. More specifically, we study the CR network in which multiple spectrum-coexisting PUs cooperate and compete with each other on their prices charged to the SUs and multiple SUs competing with each other for the limited resource are unaware of the spectrum usage scheduling of PUs. We attempt to answer the following questions: How can a

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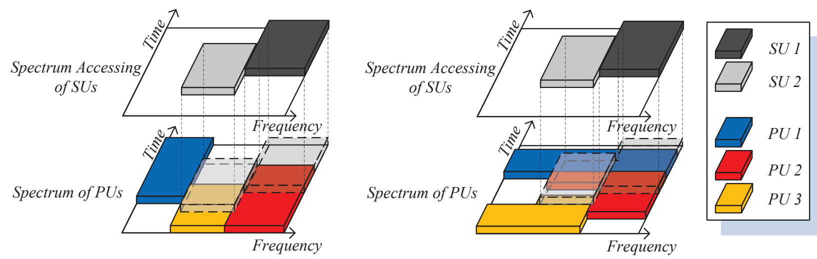

(*a*) FD-based CR networks (*b*) CR networks with spectrum pooling

Fig. 1. Comparisons of CR networks with and without spectrum pooling: In FD-based CR networks in (a), each PU owns an exclusive frequency band which can be known by SUs. If spectrum pooling is enabled in (b), multiple PUs dynamically coexist in a common spectrum pool and it is difficult for SUs to know the instantaneous transmission scheduling of PUs. We use dotted lines to denote the accessing time and frequency of the SUs in the spectrum of PUs.

flexible spectrum pooling system provide higher spectrum efficiency and better performance than the FD-based CR networks for both SU and PU systems? How can the PUs compete or cooperate with each other to maximize their payoffs? And how will the interactions of PUs affect the interactions among strategic SUs? To the best of our knowledge, this paper is the first work that tries to investigate the joint optimization problem for the spectrum pooling system with coexisting PUs and SUs.

To answer the above questions, we first propose a coalition formation game framework to study the possible price competition and cooperation among PUs and then formulate a non-cooperative game model to study the competition for transmit power and sub-band allocation among SUs. We integrate these two game theoretic models into a hierarchical game framework to study the joint optimization problem. We show that the pricing optimization problem becomes complex when multiple PUs coexist in a common spectrum pool. One of the reasons is that the transmission of each SU can be affected by the prices charged by multiple PUs. As a result, the payoff to each PU depends not only on its own price and strategy but also on those of other PUs.

We prove that allowing all PUs to fully cooperate or selfishly compete with each other is generally not the optimal choice. This is because of the diverse spatial distribution of PUs and SUs and the cost of cooperation in practical networks. This is different from most of the previous game theoretic works on wireless networks, which either neglect the cooperation costs when studying the fully cooperative model for PUs [7], [8] or assume no information exchange and coordination among PUs when investigating the fully competitive case [9]–[12]. We then focus on finding the effective methods for PUs to search for the optimal coalition formation structure. It is observed that the coalitions formed among PUs are generally overlapped. As reported in [13]–[15], finding a low-complexity algorithm for coalition formation games with overlapping coalitions is generally difficult because of the combinatorial complexity in distributing benefits to each member across multiple coalitions. In this paper, we propose a distributed coalition formation algorithm that allows PUs to form a unique, stable coalition formation. Our proposed algorithm does not require PUs to conduct an exhaustive search [16]. More importantly, the maximal number of iterations required for the proposed coalition formation algorithm does not depend on the number of PUs. In our model, each PU will refuse to join a coalition if it cannot obtain a fair share of the benefits. Therefore, investigating different fairness criteria for dividing the payoffs among PUs in each coalition is also important. We study and compare two popular fairness criteria, that is, Nash bargaining solution fairness and Shapley value fairness for PUs. Simulation results are provided to evaluate the performance of the proposed algorithm as well as the payoff division to each member under different fairness criteria.

The rest of this paper is organized as follows. The works that related to this paper is reviewed in Section II. The channel model and problem formulation are described in Section III. The game theoretic analysis and the distributed algorithm are presented in Sections IV and V, respectively. Different pricing fairness criteria are studied in Section VI. The numerical results are presented in Section VII, and the paper is concluded in Section VIII.

II. Related Works

Spectrum pooling has been recognized as one of the key technologies in next generation wireless networks [3]. A major research initiative "The End-to-End Reconfigurability (E2R)", funded by the European commission, has addressed numerous challenges in spectrum pooling [17]. The economic, policy and market challenges of spectrum pooling have been systematically investigated in [2]. The technological challenges of the possible system implementation of spectrum pooling have been surveyed in [3]. One work that is similar to spectrum pooling is the mobile virtual network operator (MVNO) system in which an MVNO can rent the spectrum and the mobile network infrastructure owned by the operator for a limit period of time [18], [19]. One of the main differences between the MVNO system and spectrum pooling is that each MVNO obtains exclusive use of spectrum and there are no overlaps between spectrum rented by different MVNOs at the same time. Another series of work, related to spectrum pooling, is presented under the framework of inter-operator spectrum sharing, which allows multiple operators to share and trade licensed spectrum with each other. More specifically, the authors in [4] proposed a distributed power allocation

TABLE I
LIST OF NOTATIONS

Symbol	Definition
$\mathcal{J}$	Set of SUs
$\mathcal{K}$	Set of PUs
$\mathcal{M}$	Set of available sub-bands
$\mathcal{L}_{S_k}$	Set of sub-bands chosen by SU S_k
$\underline{q}_{P_j}$	Charging threshold of PU P_j
$\overline{q}_{P_j}$	Interference limit of PU P_j
$\beta_{P_j[l]}$	Pricing coefficient of PU P_j in sub-band l
ϖ_{S_k}	Payoff of SU S_k
ϖ_{P_j}	Payoff of PU P_j
$\varpi_{\mathcal{J}}$	Total payoff of PUs
$w_{S_k[l]}$	Transmit power of SU S_k in sub-band l
$\mathcal{C}_{[l]}$	Set of PUs who can jointly charge the SUs in sub-band l
$\theta_{P_j[l]}\left(\mathcal{C}_{[l]}\right)$	Cooperation cost of PU P_j when it joins set $\mathcal{C}_{[l]}$ to decide the price of the SU in sub-band l

scheme for multiple operators coexisted in the same frequency band. In [20], we introduced an inter-operator carrier aggregation framework for an LTE Advanced system. A potential game theoretic model has been used in [21] to study the competition of users for base stations sharing the same frequency band. The authors in [22] developed a network architecture to achieve spectrum sharing between operators and presented simulation results to evaluate the resulting performance. In [5], the authors studied the spectrum allocation among operators sharing a common pool of spectrum resources. Different from these previous works which only focus on the interactions among PUs, in this paper, we consider joint optimization problem for a spectrum pooling system with both SUs and PUs.

III. SYSTEM MODEL AND BASIC GAME SETUP

A. Channel Model

Let the sets of J PUs and K SUs be $\mathcal{J} = \{P_1, P_2, \ldots, P_J\}$ and $\mathcal{K} = \{S_1, S_2, \ldots, S_K\}$, respectively. We assume that the SUs use orthogonal frequency division multiple access (OFDMA)[1], i.e., each sub-band can only be occupied by one SU. This can be achieved by using proper multiple access protocols [23], [24] in SU networks. All the SUs compete for the set of the available sub-bands $\mathcal{M} = \{1, 2, \ldots, M\}$ with $M \geq K$. We assume that each SU S_k can use multiple sub-bands, and we label the set of sub-bands of SU S_k as $\mathcal{L}_{S_k} \subseteq \mathcal{M}$ for $\mathcal{L}_{S_k} \cap \mathcal{L}_{S_j} = \emptyset$. Let $\boldsymbol{\mathcal{L}}_S = [\mathcal{L}_{S_1}, \mathcal{L}_{S_2}, \ldots, \mathcal{L}_{S_K}]$ be the sub-band allocation scheme for the K SUs. The PUs and SUs have no a priori knowledge of each other's signal information.

In a practical system, any transmission of SUs can always generate interference to the PU network. However, the PUs can only charge SUs for the use of their licensed spectrum if the interference caused by the SUs is "noticeable". In this paper, we assume that each PU only charges an SU if the received interference from this SU is higher than a threshold, called the *charging threshold*. Let us define the charging threshold for P_j as $\underline{q}_{P_j}$, i.e., P_j can only charge the presence of an SU S_k in sub-band l if

$$h_{jk[l]} w_{S_k[l]} > \underline{q}_{P_j}, \tag{1}$$

where $h_{jk[l]}$ is the ratio of channel gain between S_k and P_j in sub-band l to the additive interference and noise power received at PU P_j and $w_{S_k[l]}$ is the transmit power of S_k in sub-band l for $l \in \mathcal{M}$. It can be easily observed from (1) that the charging threshold is exactly the same as the *SNR wall* proposed in [25] if every PU uses energy detection to detect the existence of interfering SUs. In a spectrum pooling system, all PUs combine their licensed spectrum to form a common spectrum pool and hence each PU should be able to charge and decide the prices of the SUs that access the common pool. Our model can be applied into most of the other previously studied scenarios in the literature. For example, if only one PU can charge all the sub-bands, i.e., $h_{jk[l]} = 0 \ \forall \ l \in \mathcal{M}$, $j \in \{2, 3, \ldots, J\}$ and $k \in \{1, 2, \ldots, K\}$, the system model under our consideration is equivalent to the CR network with one PU (i.e., PU P_1) and multiple SUs [26], [27]. If we assume $J = M$ and each PU can only charge one sub-band of SUs, i.e., $h_{jk[l_{S_k} \neq l_{P_j}]} = 0$ for $j \neq k$ (P_j only uses the sub-band l_{P_j}), the system model under our consideration becomes the FD-based CR network in which PUs use FD mode to send their signals and the SUs observe independent PU's actions/states in each sub-band [28], [29]. If we assume $|\mathcal{L}_{S_k}| = 1$ for $S_k \in \mathcal{K}$, our model becomes the system in which each SU can only access one sub-band.

For SUs, (1) means that an SU S_k can use the licensed spectrum for free if its transmit power is low enough to satisfy the regularity. Therefore, (1) can be regarded as a lower bound on the transmit powers for the SUs that are "affected" by the prices of the PUs. In other words, PUs can only charge for the presence of an SU when the received interference caused by the SU is larger than the threshold [30]. In this paper, we neglect the unchargeable SUs and only focus on the SUs that are charged by at least one PU. In other words, the set of sub-bands allocated to each SU is a subset of all the sub-bands satisfying the transmit power constraint in (1), i.e., $\mathcal{L}_{S_k} \subseteq \{l : h_{jk[l]} w_{S_k[l]} > \underline{q}_{P_j}, \ \forall l \in \mathcal{M}\}$.

Another constraint for the SUs is the *interference limit* of PUs. PUs in the spectrum pooling system can access any parts of the spectrum pool and hence need to make sure that their transmissions in each portion of the spectrum satisfy a certain level of quality of service (QoS). We hence assume that each PU P_j imposes the maximum tolerable interference limit $\overline{q}_{P_j}$ in each sub-band for every SU, i.e., $h_{jk[l]} w_{S_k[l]} < \overline{q}_{P_j}$, $\forall \ l \in \mathcal{M}$ and $P_j \in \mathcal{J}$. This constraint also satisfies the average interference limit of each PU P_j for the entire licensed spectrum, i.e., $\frac{1}{KM} \sum_{S_k \in \mathcal{K}} \sum_{l \in \mathcal{M}} h_{jk[l]} w_{S_k[l]} \leq \overline{q}^{\mathrm{ave}}_{P_j}$, or total interference limit, i.e., $\sum_{S_k \in \mathcal{K}} \sum_{l \in \mathcal{M}} h_{jk[l]} w_{S_k[l]} \leq \overline{q}^{\mathrm{tot}}_{P_j}$. Hence, the power

[1]We consider the OFDMA system to simplify our discussion. Our results can be easily extended into other systems with other communication modes. Specifically, each sub-band can correspond to an exclusive resource block, e.g., time, antenna, frequency etc.

constraint for an SU S_k in each sub-band l is defined as

$$w_{S_k[l]} \le \min_{P_j \in \mathcal{J}} \left\{ \frac{\overline{q}_{P_j}}{h_{jk[l]}} \right\}. \tag{2}$$

In this paper, we assume that $\overline{q}_{P_j} > \underline{q}_{P_j}$, $\forall P_j \in \mathcal{J}$. The list of major notations used in this paper is provided in Table I.

B. Hierarchical Game Setup

We introduce a hierarchical game theoretic framework in which the players of the game include both PUs and SUs. The PUs have the priority in using the spectrum and the SUs try to access the licensed spectrum and by paying prices to the PUs.

We assume that if a PU is charging an SU S_k in a sub-band, the PU can detect whether this SU has also been charged by other PUs by eavesdropping whether any other pricing signals are sent in the same sub-band. Note that each PU can only detect whether or not there are any other PUs that charge the same SU. However, this PU cannot know the exact prices of other PUs or how many PUs are charging SU S_k, e. g., the pricing function of each PU may be encrypted. Each PU will only send unencrypted pricing information to the PUs it tries to cooperate with. Let the subset of PUs who charge the SU in sub-band l be $\mathcal{C}_{[l]}$ and the set of sub-bands that are charged by P_j be $\mathcal{L}_{P_j}$. We define the payoff of SU S_k in a sub-band $l \in \mathcal{L}_{S_k}$ as

$$\varpi_{S_k[l]} \left(w_{S_k[l]}, \boldsymbol{\beta}_{[l]}, \boldsymbol{\mathcal{L}}_S \right) = R_{S_k[l]} - \boldsymbol{\beta}_{[l]} \boldsymbol{h}_{\bullet k[l]} w_{S_k[l]}, \tag{3}$$

where $R_{S_k[l]} = \log_2 \left(1 + g_{S_k[l]} w_{S_k[l]}\right)$, $g_{S_k[l]}$ is the ratio of the channel gain between the kth secondary sender-to-receiver pair to the additive interference and noise power received by SU S_k in sub-band l[2]. $\boldsymbol{\beta}_{[l]} = (\beta_{P_j[l]})_{P_j \in \mathcal{C}_{[l]}}$ is a $|\mathcal{C}_{[l]}| \times 1$ vector, $\beta_{P_j[l]}$ is the pricing coefficient of PU P_j charged to the SU in sub-band l. $\boldsymbol{h}_{\bullet k[l]} = \left(h_{jk[l]}\right)^{\dagger}_{P_j \in \mathcal{C}_{[l]}}$ and $\dagger$ denotes the transpose of a matrix. The overall payoff of SU S_k can then be written as

$$\varpi_{S_k} \left(\boldsymbol{w}_{S_k}, \boldsymbol{\beta}, \boldsymbol{\mathcal{L}}_S \right) = \sum_{l \in \mathcal{L}_{S_k}} \varpi_{S_k[l]} \left(w_{S_k[l]}, \boldsymbol{\beta}_{[l]}, \boldsymbol{\mathcal{L}}_S \right), \tag{4}$$

where $\boldsymbol{w}_{S_k} = \left(w_{S_k[l]}\right)_{l \in \mathcal{L}_{S_k}}$, $\boldsymbol{\beta} = \left(\beta_{P_j[l]}\right)_{P_j \in \mathcal{J}, l \in \mathcal{M}}$ is a $|\mathcal{J}| \times |\mathcal{M}|$ matrix.

In our model, SUs are selfish and seek a sub-band allocation scheme that no SU could benefit by unilaterally changing its allocated sub-bands, i.e., each SU S_k tries to search for a set $\mathcal{L}^*_{S_k}$ of sub-bands by solving the following problem,

$$\mathcal{L}^*_{S_k} = \arg \max_{\mathcal{L}_{S_k} \subseteq \mathcal{M}} \varpi_{S_k} \left(\boldsymbol{w}_{S_k}, \boldsymbol{\beta}, \mathcal{L}_{S_k}, \boldsymbol{\mathcal{L}}^*_{-S_k} \right), \tag{5}$$

where $-S_k$ denotes all SUs except S_k. In addition, each SU S_k can adopt the optimal power control by solving the following problem in each sub-band $l \in \mathcal{L}_{S_k}$

$$w^*_{S_k[l]} \left(\boldsymbol{\beta}_{[l]} \right) = \arg \max_{w_{S_k[l]}} \varpi_{S_k[l]} \left(w_{S_k[l]}, \boldsymbol{\beta}_{[l]}, \boldsymbol{\mathcal{L}}^*_S \right). \tag{6}$$

[2]In this paper, we assume the interference caused by PUs to the SUs can be regarded as a part of the additive noise at the corresponding receiver of each SU. In other words, $g_{S_k[l]}$ is actually the ratio of SU S_k's channel gain to the aggregated additive noise and the interference from PUs.

Let us define the payoff obtained in each sub-band l and the total payoff of each PU P_j as

$$\varpi_{P_j[l]}(\boldsymbol{w}_S, \boldsymbol{\beta}_{[l]}, \boldsymbol{\mathcal{L}}_S, \mathcal{C}_{[l]}) = R_{P_j[l]} - \theta_{P_j[l]} \left(\mathcal{C}_{[l]} \right), \tag{7}$$

$$\varpi_{P_j}(\boldsymbol{w}_S, \boldsymbol{\beta}, \boldsymbol{\mathcal{L}}_S, \boldsymbol{\mathcal{C}}) = \sum_{l \in \mathcal{L}_{P_j}} \varpi_{P_j[l]}(\boldsymbol{w}_S, \boldsymbol{\beta}_{[l]}, \boldsymbol{\mathcal{L}}_S, \mathcal{C}_{[l]}) \tag{8}$$

where $\boldsymbol{w}_S = (\boldsymbol{w}_{S_k})_{S_k \in \mathcal{K}}$, $\boldsymbol{\mathcal{C}} = \left(\mathcal{C}_{[l]}\right)_{l \in \mathcal{M}}$, $\boldsymbol{\beta}_{P_j} = \left(\beta_{P_j[l]}\right)_{l \in \mathcal{L}_{P_j}}$ and $R_{P_j[l]} = \beta_{P_j[l]} \sum_{S_k \in \mathcal{K}} \mathbf{1}_{l \in \mathcal{L}_{S_k}} h_{jk[l]} w_{S_k[l]}$ is the revenue obtained by PU P_j from charging the SU in sub-band l and $\mathbf{1}_{l \in \mathcal{L}_{S_k}}$ is the indicator function, i.e., if $l \in \mathcal{L}_{S_k}$ (or $l \notin \mathcal{L}_{S_k}$), then $\mathbf{1}_{l \in \mathcal{L}_{S_k}} = 1$ (or $\mathbf{1}_{l \in \mathcal{L}_{S_k}} = 0$). $\theta_{P_j[l]} \left(\mathcal{C}_{[l]} \right) \ge 0$ is the cooperation cost of P_j when it joins a set $\mathcal{C}_{[l]}$ to charge the SU in sub-band l. $\theta_{P_j[l]} \left(\mathcal{C}_{[l]} \right) = 0$ if P_j does not belong to any sets, i.e., $\mathcal{C}_{[l]} = \emptyset$, or is the only member in a set to charge an SU, i.e., $\mathcal{C}_{[l]} = \{P_j\}$. If a PU P_j is involved in a multiple-PU set, i.e., $|\mathcal{C}_{[l]}| \ge 2$, then $\theta_{P_j[l]} \left(\mathcal{C}_{[l]} \right)$ should be a positive value related to the transmit power and other resources, such as time, spectrum, etc., used in exchanging cooperation-related information among member PUs in the set $\mathcal{C}_{[l]}$ [16]. In our model, for any two disjoint subsets $\mathcal{C}^1$ and $\mathcal{C}^2$, we have $\sum_{P_j \in \mathcal{C}^1 \cup \mathcal{C}^2} \theta_{P_j}(\mathcal{C}^1 \cup \mathcal{C}^2) > \sum_{P_j \in \mathcal{C}^1} \theta_{P_j}(\mathcal{C}^1) + \sum_{P_j \in \mathcal{C}^2} \theta_{P_j}(\mathcal{C}^2)$. The value of $\theta_{P_j[l]} \left(\mathcal{C}_{[l]} \right)$ directly affects the willingness of PU P_j to join a set $\mathcal{C}_{[l]}$. If $\theta_{P_j[l]}(\mathcal{C}_{[l]})$ exceeds the maximum revenue that P_j could obtain by joining a set $\mathcal{C}_{[l]}$, P_j will not cooperate with any PU to set the price charged to the SU in sub-band l. As we will observe later, the sets of PUs to charge different SUs may be overlapped with each other. Hence, the cooperation costs for a PU to join the sets charging different SUs may be correlated with each other. For instance, if PU P_j cooperates with another PU $P_{j'}$ to charge an SU S_k in sub-band l, the cooperation cost between the same PUs P_j and $P_{j'}$ for another SU $S_{k'}$ in another sub-band l' may not be as expensive as that between PU P_j and a different PU $P_{j''}$ for $j'' \ne j, j'' \ne j'$ and $k' \ne k$.

In this paper, we study the pricing competition and cooperation for the PUs to maximize their payoffs. More specifically, in our model, each PU can either define its price independently without communicating with other PUs, or negotiate with other PUs to jointly define the prices charged to SUs in every sub-band. That is, the optimization problem for each PU P_j in sub-band l is given by

$$\max_{\mathcal{C}_{[l]}, \beta_{P_j[l]}} \varpi_{P_j[l]} \left(\boldsymbol{w}^*_S, \beta_{P_j[l]}, \boldsymbol{\beta}_{-P_j[l]}, \boldsymbol{\mathcal{L}}^*_S, \mathcal{C}_{[l]} | \mathcal{F}_{\mathcal{C}_{[l]}} \right). \tag{9}$$

where $\mathcal{F}_{\mathcal{C}_{[l]}}$ is the fairness criterion agreed upon by all the PUs in set $\mathcal{C}_{[l]}$.

To solve the above problem, we develop a coalitional game model in which all the PUs in one coalition only care about the payoff sum. This payoff sum could then be divided among all the members according to an agreed fairness criterion. We hence can regard the PUs in one coalition $\mathcal{J}$ as a single entity with payoff $\varpi_{\mathcal{J}} \left(\boldsymbol{w}^*_S, \boldsymbol{\beta}, \boldsymbol{\mathcal{L}}_S, \boldsymbol{\mathcal{C}} \right) = \sum_{P_j \in \mathcal{J}} \varpi_{P_j} \left(\boldsymbol{w}^*_S, \boldsymbol{\beta}, \boldsymbol{\mathcal{L}}_S, \boldsymbol{\mathcal{C}} \right)$.

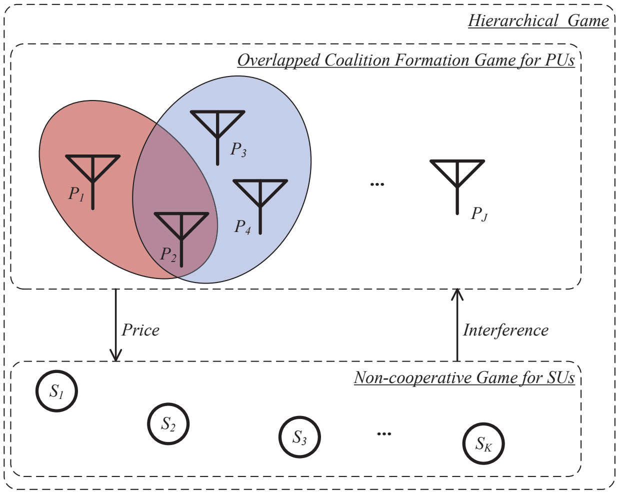

Fig. 2. The relationship of different games in the proposed model of this paper.

We then introduce a hierarchical game framework to find an equilibrium point, known as the Stackelberg Equilibrium (SE), for both SUs and PUs. The SE is defined as follows[3].

Definition 1: [31, Definitions 3.27] We say $\boldsymbol{\beta}^* = \left(\beta^*_{P_j[l]}\right)_{l\in\mathcal{M},P_j\in\mathcal{J}}$ and $\boldsymbol{w}^*_S = \left(w^*_{S_k[l]}\left(\boldsymbol{\beta}^*_{[l]}\right)\right)_{S_k\in\mathcal{K},l\in\mathcal{M}}$ form an SE with PUs as the leaders and SUs as the followers for a given sub-band allocation scheme $\boldsymbol{\mathcal{L}}_S$ if the power constraints in (1) and (2) are satisfied, and for PUs, we have

$$\boldsymbol{\beta}^*_{P_j} = \arg\max_{\boldsymbol{\beta}_{P_j}\geq 0} \varpi_{\mathcal{J}}\left(\boldsymbol{w}^*_S, (\boldsymbol{\beta}_{P_j}, \boldsymbol{\beta}^*_{-P_j}), \boldsymbol{\mathcal{L}}_S, \boldsymbol{\mathcal{C}}\right)\ \forall\ P_j \in \mathcal{J} \quad (10)$$

where $\boldsymbol{\beta}^*_{-P_j} = \left(\boldsymbol{\beta}^*_{P_i[l]}\right)_{l\in\mathcal{M},P_i\in\mathcal{J}\setminus P_j}$ and for a given $\boldsymbol{\beta}^*$, $\boldsymbol{w}^*_{S_k}\left(\boldsymbol{\beta}^*\right)$ is given by

$$\boldsymbol{w}^*_{S_k}(\boldsymbol{\beta}^*) = \arg\max_{\boldsymbol{w}_{S_k}\geq 0} \varpi_{S_k}\left(\boldsymbol{w}_{S_k}, \boldsymbol{\beta}^*, \boldsymbol{\mathcal{L}}_S\right). \quad (11)$$

We say a pair $(\boldsymbol{\beta}^*, \boldsymbol{w}^*_S)$ is the *Pareto optimal SE* if $\boldsymbol{\beta}^*$ maximizes the payoff sum of PUs and $\boldsymbol{w}^*_S$ maximizes the payoff sum of SUs.

IV. Game Theoretic Analysis

In this section, we first derive the SE of the hierarchical game model in Section IV-A. Then, in Section IV-B, we show that neither fully competition nor fully cooperation among PUs obtains optimal solutions and hence it is necessary to focus on optimizing the possible pricing coalition formation structure among PUs.

[3]Note that in [31, Definitions 3.27], the leader will choose the strategy that can greentree its payoff in the worst case. However, as will shown later in this paper, in our game, for the give pricing coefficients of PUs (leaders) and sub-band allocation scheme $\boldsymbol{\mathcal{L}}_S$, each SU (follower) will choose a unique optimal transmit power to transmit its signal. Therefore, we can write the SE of our game in (10) in which each leader will search for its strategy of SE by assuming all SUs will use the optimal transmit power defined in (11). Note that in our proposed algorithm in Section IV, PUs do not require to know the optimal transmit powers of the SUs.

A. Hierarchical Game Theoretic Analysis

Each SU can optimize its transmit power and sub-band access according to the given pricing coefficients of PUs. By assuming $\boldsymbol{\beta}$, $\boldsymbol{\mathcal{C}}$ and $\boldsymbol{\mathcal{L}}_S$ to be fixed, we can solve (6) and obtain the optimal transmit power of each SU S_k in every sub-band $l \in \mathcal{L}_{S_k}$ in equation (12) at the top of next page, where $(x)^+ = \max\{x, 0\}$, $u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}, \mathcal{C}_{[l]}\right) = \boldsymbol{\beta}_{[l]}\boldsymbol{h}_{\bullet k[l]}$. The above result can be regarded as the optimal transmit powers of SUs [12] with power constraints of the spectrum pool defined in (1) and (2). Note that, if $w^*_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}\right) = \min_{P_j\in\mathcal{J}}\left\{\frac{q_{P_j}}{h_{jk[l]}}\right\}$, the resulting interference of S_k is too low for S_k to be charged by any PUs in sub-band l. In this case, the generated interference by SU S_k in sub-band l is lower than the charging thresholds of all the PUs. If $\mathcal{C}_{[l]} \neq \emptyset$, the transmit power of at least one SU must be positive and satisfy the power constraint in (1) in sub-band l. We have the following result about the SE of the proposed hierarchical game with a given sub-band allocation scheme $\boldsymbol{\mathcal{L}}_S$.

Proposition 1: Assume $\boldsymbol{\mathcal{C}}$ and $\boldsymbol{\mathcal{L}}_S$ are fixed. $(\boldsymbol{w}^*_S, \boldsymbol{\beta}^*)$ is a pure strategy SE if the following equality holds,

$$u_{S_k[l]}\left(\boldsymbol{\beta}^*_{[l]}, \mathcal{C}_{[l]}\right) = \frac{1}{\min_{P_i\in\mathcal{C}_{[l]}}\left\{\frac{\overline{q}_{P_i}}{h_{ik[l]}}\right\} + \frac{1}{g_{S_k[l]}}}, \quad \forall l \in \mathcal{L}_{S_k} \text{ and } S_k \in \mathcal{K}, \quad (13)$$

and $\boldsymbol{w}^*_S = \left(w^*_{S_k[l]}\right)_{S_k\in\mathcal{K},l\in\mathcal{M}}$ where $w^*_{S_k[l]}$ is given in (12).

Proof: See Appendix A. ■

From the above proposition and proof in Appendix A, we can observe that the strategic SUs (each SU selects its optimal transmit power in (12)) force the PUs to decrease their prices to achieve the optimal payoffs. Note that, if the SUs use the fixed transmit powers, the PUs should always charge the highest possible prices to all SUs to maximize their payoffs.

It is observed that the SE of the hierarchical game is closely related to the sub-band allocation scheme $\boldsymbol{\mathcal{L}}_S$ of SUs [12]. More specifically, each sub-band allocation scheme corresponds to a different set of SEs. Let us define a *sub-band allocation game* whose *players* are all the SUs, payoff of each play is the payoff defined in (4) and the *action* of each player is to choose a set of sub-bands that maximize its payoff given the sub-bands allocated to other SUs. In Section IV, we will propose a simple algorithm that can achieve a unique Nash equilibrium (NE) [32, Definition 21.1] of the sub-band allocation game for SUs.

B. Coalition Formation Game Theoretic Analysis for PUs

First, let us consider the case that all the PUs selfishly compete with each other on the prices charged to the SUs and assume that all the PUs cannot exchange any information. As observed previously. Because of the spectrum coexistence of PUs, the optimal price of each PU depends on the strategies and prices of other PUs. In other words, it is generally impossible for all the PUs to choose the optimal pricing coefficients to maximize their payoff sum

$$w^{*}_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}\right)=\begin{cases}\min\limits_{P_j\in\mathcal{J}}\left\{\frac{\underline{q}_{P_j}}{h_{jk[l]}}\right\}, & \text{if } u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]},\mathcal{C}_{[l]}\right)\geq\frac{1}{\min\limits_{P_j\in\mathcal{J}}\left\{\frac{\underline{q}_{P_j}}{h_{jk[l]}}\right\}+\frac{1}{g_{S_k[l]}}},\\ \left(\frac{1}{u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]},\mathcal{C}_{[l]}\right)}-\frac{1}{g_{S_k[l]}}\right)^{+}, & \text{if } \frac{1}{\min\limits_{P_j\in\mathcal{J}}\left\{\frac{\underline{q}_{P_j}}{h_{jk[l]}}\right\}+\frac{1}{g_{S_k[l]}}}>u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]},\mathcal{C}_{[l]}\right)>\frac{1}{\min\limits_{P_i\in\mathcal{J}}\left\{\frac{\overline{q}_{P_i}}{h_{ik[l]}}\right\}+\frac{1}{g_{S_k[l]}}},\\ \min\limits_{P_j\in\mathcal{J}}\left\{\frac{\overline{q}_{P_j}}{h_{jk[l]}}\right\}, & \text{if } u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]},\mathcal{C}_{[l]}\right)\leq\frac{1}{\min\limits_{P_i\in\mathcal{J}}\left\{\frac{\overline{q}_{P_i}}{h_{ik[l]}}\right\}+\frac{1}{g_{S_k[l]}}},\end{cases}\quad(12)$$

without exchanging information with each other. We hence have the following results.

Observation 1: If PUs cannot exchange any information among each other, the SE of the hierarchical game is not optimal if there exist two PUs P_j and P_i such that $\underline{q}_{P_j} < \overline{q}_{P_i}$ and the cost of cooperation between P_j and P_i is negligible for $i \neq j$ and $i, j \in \{1, 2, \ldots, J\}$.

Proof: See Appendix B. ■

Let us now introduce the concept of the coalitional game and consider another extreme case in which all the PUs fully cooperate with each other and jointly decide the prices for every SU in every sub-band. We will prove in Theorem 2 that this case is also not optimal for the PUs. Let us provide the following definitions that are useful in our analysis.

Definition 2: [13, Chapter 9] We define the set of all the players as the *grand coalition* $\mathcal{J}$. A *coalition* $\mathcal{C}$ is a nonempty subset of the grand coalition $\mathcal{J}$. A *coalitional game* is defined by $(\mathcal{J}, v)$ where v is the *characteristic function*, which associates every coalition $\mathcal{C}$ with a number $v(\mathcal{C})$. Here, $v(\mathcal{C})$ is called *worth* which in this paper is equivalent to the total payoff of a coalition $\mathcal{C}$. We have $v(\emptyset) = 0$.

Definition 3: A coalitional game is said to be *super-additive* if for any two disjoint coalitions $\mathcal{C}^1$ and $\mathcal{C}^2$, for $\mathcal{C}^1, \mathcal{C}^2 \subset \mathcal{J}$, we have $v\left(\mathcal{C}^1 \cup \mathcal{C}^2\right) \geq v\left(\mathcal{C}^1\right) + v\left(\mathcal{C}^2\right)$.

Definition 4: A *payoff vector* of the coalition $\mathcal{J}$ is any vector $\boldsymbol{\varpi} = (\varpi_{P_j})_{P_j \in \mathcal{J}}$ in $\mathbb{R}^J$ to divide the value $v(\mathcal{J})$. $\boldsymbol{\varpi}$ is said to be *group rational* or *efficient* if $\sum_{j=1}^{J} \varpi_{P_j} = v(\mathcal{J})$ and is said to be *individual rational* if $\varpi_{P_j} \geq v(\{P_j\}), \forall P_j \in \mathcal{J}$. Let us also define an *imputation* as a payoff vector satisfying both group and individual rationality.

Definition 5: An imputation $\boldsymbol{\varpi}$ is *unstable* through a coalition $\mathcal{C}$ if $v(\mathcal{C}) > \sum\limits_{P_j \in \mathcal{C}} \varpi_{P_j}$. The *core* of $v(\mathcal{J})$ is defined as the set of stable imputations. $\boldsymbol{\varpi}$ is in the core if and only if

$$\sum_{P_j \in \mathcal{J}} \varpi_{P_j} = v(\mathcal{J}) \text{ and } \sum_{P_j \in \mathcal{C}} \varpi_{P_j} \geq v(\mathcal{C}), \forall \mathcal{C} \subseteq \mathcal{J}. \quad (14)$$

Recall that, in our model, each PU can only charge the SUs whose resulting interference is larger than the charging threshold. As we observe in the proof of Proposition 1, if a PU P_j refuses to join a coalition $\mathcal{C}_{[l]}$, then it can always raise its pricing coefficient $\beta_{P_j[l]}$ and will not allow SU S_k to cause chargeable interference to its transmission. More specifically, if $l \notin \mathcal{L}_{P_j}$, PU P_j will raise its pricing coefficient to limit the transmit power of SU S_k in sub-band l to be lower than its charging threshold, i.e., $w_{S_k[l]}$ should satisfy $h_{jk[l]} w_{S_k[l]} < \underline{q}_{P_j}$.

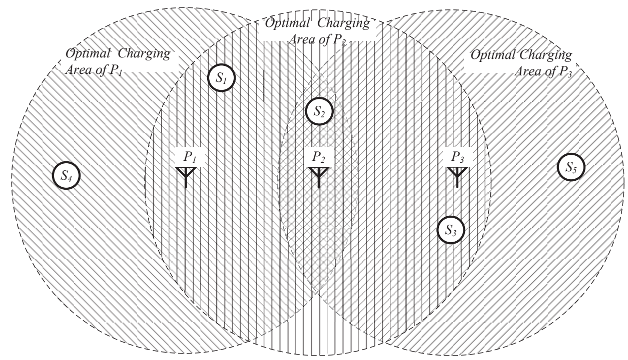

Fig. 3. An example CR network with five SUs and three PUs. The shadowed areas denote the optimal charging areas (SUs within this area should be charged by the corresponding PU) of each PU.

We have the following results about the stability of the grand coalition for our pricing coalitional game.

Observation 2: The core of the grand coalition $\mathcal{J}$ of the pricing coalitional game is empty if there exists at least one $P_j \in \mathcal{J}$ that satisfies the following condition in one sub-band l,

$$R_{P_j[l]} < \theta_{P_j[l]}(\mathcal{J}). \quad (15)$$

Proof: See Appendix C. ■

Note that the condition in (15) may be satisfied by the situations that the channel gain variations and interference limits of other PUs cause the transmit power of the SU S_k that accesses sub-band l to be lower than the chargeable threshold of P_j, or the cost for P_j to join the grand coalition $\mathcal{J}$ to be larger than the benefits obtained from charging SU S_k in sub-band l.

We have the following remark from the above observations.

Remark 1: The core of the grand coalition is always empty in a large multi-user CR network.

The above remark follows from the observation that the condition in (15) can usually be satisfied for a CR network. Let us illustrate this through an example shown in Figure 3. Suppose that all SUs are randomly distributed in a network. To simplify our discussion, we assume that each SU has been allocated with an exclusive sub-band and hence neglect the labels of the specific sub-band used by each SU will not cause any confusion. Three PUs with equal interference limits are located in a linear network. In this case, allowing PU P_3 to cooperate with PU P_1 to decide the price of SU S_4 may not be the optimal choice because allowing the transmit power of SU S_4 to be larger than the charging threshold of PU P_3 may result in a higher-than-tolerable interference for PU P_1. In addition, because the distance between PUs P_3 and P_1 is large, the cooperation cost for forming a coalition is high too. Even the transmit power of SU S_4 does not cause high interference at PU P_1, the long distance between PUs P_1 and P_3 may result in excessive energy consumption and large delay during their information exchange. Also, if the cooperation costs of PU P_3 to charge SUs S_2 and S_3 are larger than the benefits obtained from SUs S_2, S_3 and S_5, then PU P_3 will have no incentive to join the grand coalition but will only form a coalition with PU P_2 to charge SUs S_2, S_3 and S_5. Thus, it can be observed that PU P_3 should only charge prices to S_5 and form a coalition with P_2 to charge SUs S_3 and also cooperate with both PUs P_1 and P_2 to charge SU S_2. And P_3 should not join any coalitions in deciding the prices of the farthest SUs S_1 and S_4. The above observation can be easily extended to a general CR network.

V. A Distributed Joint Optimization Algorithm

We now consider the possible pricing coalition formation among PUs. Let us illustrate an example using Figure 3 again to show the characteristics of coalition formation among PUs. First of all, it is observed that the coalitions formed by PUs to decide the prices of different SUs may not be independent. For example, in Figure 3, PU P_2 should cooperate with PU P_1 on deciding the price charging to SU S_1 in sub-band l and also cooperate with PU P_3 on choosing the price charged to SU S_3 in another sub-band l'. Our second observation is that the cooperation between two disjoint coalitions may not always improve the payoff sum. More specifically, let us consider the coalition formation process among three PUs to charge an SU S_k in sub-band l. Assume that the channel gains between SU S_2 and three PUs (P_2, P_1 and P_3) in sub-band l satisfy $\frac{q_{P_2}}{h_{22[l]}} < \frac{q_{P_1}}{h_{12[l]}} < \frac{q_{P_3}}{h_{32[l]}}$, and $\max\limits_{j\in\{1,2,3\}} \left\{ \frac{q_{P_j}}{h_{j2[l]}} \right\} < \min\limits_{j\in\{1,2,3\}} \left\{ \frac{q_{P_j}}{h_{j2[l]}} \right\}$. To avoid confusions caused by the cooperation cost, we consider the case that the cooperation cost does not play a dominant role in the coalition formation process and hence if forming a coalition between any two disjoint subsets improves the total revenue of the PUs, it will also improve their total payoff. That is, if $R_{\mathcal{C}^1_{[l]} \cup \mathcal{C}^2_{[l]}} > R_{\mathcal{C}^1_{[l]}} + R_{\mathcal{C}^2_{[l]}}$, then $v\left(\mathcal{C}^1_{[l]} \cup \mathcal{C}^2_{[l]}\right) > v\left(\mathcal{C}^1_{[l]}\right) + v\left(\mathcal{C}^2_{[l]}\right)$ where $R_{\mathcal{C}} = \sum\limits_{P_j \in \mathcal{C}_{[l]}} R_{P_j[l]}$ and $\mathcal{C}^1_{[l]}$ and $\mathcal{C}^2_{[l]}$ are any two disjoint subsets of $\{P_1,P_2,P_3\}$. If no cooperation is allowed among PUs, as observed in Observation 1 and Appendix B, the payoffs of three PUs obtained from charging SU S_2 in sub-band l are given by $\varpi_{P_2[l]} = \beta_{P_2[l]} h_{22} \frac{q_{P_1}}{h_{12[l]}}$, $\varpi_{P_1[l]} = \varpi_{P_3[l]} = 0$. If we allow PUs P_1 and P_2 to form a coalition $\{P_1, P_2\}$ to charge SU S_2, the payoff sum of these two cooperative PUs becomes $\sum\limits_{i\in\{1,2\}} \varpi_{P_i[l]} = \sum\limits_{i\in\{1,2\}} \left(\frac{\beta_{P_i[l]} h_{i2} q_{P_3}}{h_{32[l]}} - \theta_{P_i[l]}\left(\{P_1, P_2\}\right) \right)$ which is always larger than the sum of their payoffs without cooperations. However, this result does not hold when P_2 and P_3 cooperate without P_1. In this case, the payoff sum is $\sum\limits_{i\in\{2,3\}} \varpi_{P_i[l]} = \beta_{P_2[l]} h_{22[l]} \frac{q_{P_1}}{h_{12[l]}} - \sum\limits_{i=\{2,3\}} \theta_{P_i}\left(\{P_2, P_3\}\right)$, which is always lower than the payoff sum without cooperation because of cost of cooperation. To sum up, the coalition formation framework of our model is different from the traditional coalitional game model in [7], [16] in the following senses:

1) The coalitions formed among PUs to charge different SUs may be overlapped,
2) Cooperation between two disjoint coalitions of PUs does not necessarily increase the payoff sum.

To solve the first issue, let us convert all the overlapped coalitions into independent ones as follows. It is observed in Section III-B that the payoff function of each PU P_j in (8) is given by the summation of its payoff functions charged to all of its chargeable SUs. It is observed that maximizing the payoff of each PU P_j is equivalent to maximizing the payoff of PU P_j earned from every sub-band charged to every chargeable SU. We hence can separate the payoff of the PU P_j into different independent parts according to different SUs. In this way, for the rest of this paper, we only need to focus on a pricing coalitional game in one frequency band l in which PUs in a set $\mathcal{C}_{[l]}$ cooperate with each other in deciding the price charged to an SU S_k.

To solve the second issue, we rearrange the labeling sequence of the PUs in $\mathcal{C}_{[l]}$ by $\{P_{\tilde{1}},\ P_{\tilde{2}}, \ldots,\ P_{\widetilde{|\mathcal{C}_{[l]}|}}\}$ where $\frac{q_{P_{\widetilde{j-1}}}}{h_{\widetilde{j-1}k[l]}} < \frac{q_{P_{\tilde{j}}}}{h_{\tilde{j}k[l]}} < \frac{q_{P_{\widetilde{j+1}[l]}}}{h_{\widetilde{j+1}k}}$, $\forall\ j \in \left\{2, 3, \ldots, |\mathcal{C}_{[l]}| - 1\right\}$. We say the PUs are *sequential* if their rearranged labels are consecutive, i.e., $P_{\widetilde{j-1}}, P_{\tilde{j}}, \ \ldots,\ P_{\widetilde{j+l}}$ is sequential. We say one set is *sequential* if all the elements in this set are sequential. We say two or more disjoint sets are *sequential* if each of these sets are sequential and the union of these sets is sequential too, $\mathcal{C}^1 = \{P_{\tilde{1}}, \ldots, P_{\tilde{l}}\}$ and $\mathcal{C}^2 = \{P_{\widetilde{l+1}}, \ldots, P_{\tilde{j}}\}$ for $1 < l < j$ are sequential. We denote the set of all the possible sequential sub-sets of $\mathcal{C}$ as $\widetilde{\mathcal{C}}$.

We have the following property for the proposed game.

Proposition 2: Assume (2) is always satisfied. Suppose two disjoint nonempty coalitions $\mathcal{C}^1_{[l]}$ and $\mathcal{C}^2_{[l]}$ for $\mathcal{C}^1_{[l]}, \mathcal{C}^2_{[l]} \subset \mathcal{C}_{[l]}$ satisfy the following conditions,

1) $P_{\tilde{1}} \in \mathcal{C}^1_{[l]} \cup \mathcal{C}^2_{[l]}$,
2) $\mathcal{C}^1_{[l]} \cup \mathcal{C}^2_{[l]}$ is sequential,
3) If $R_{\mathcal{C}^1_{[l]} \cup \mathcal{C}^2_{[l]}} > \sum\limits_{n\in\{1,2\}} R_{\mathcal{C}^n_{[l]}}$, we have $v\left(\mathcal{C}^1_{[l]} \cup \mathcal{C}^2_{[l]}\right) > \sum\limits_{n\in\{1,2\}} v\left(\mathcal{C}^n_{[l]}\right)$.

Then, $\mathcal{C}^1_{[l]}$ and $\mathcal{C}^2_{[l]}$ satisfy the super-additive condition.

Proof: See Appendix D. ■

The constrained coalitional game with all the subsets of member PUs of a coalition $\mathcal{C}_{[l]}$ satisfying the above conditions is referred to as a *sequential coalitional game*.

Before presenting our proposed coalition formation algorithm, let us provide some definitions which are useful for proving our results.

Definition 6: An (overlapped) *coalition formation structure* in the grand coalition $\mathcal{J}$ is any arbitrary group of coalitions

$\{\mathcal{C}^1, \mathcal{C}^2, \dots, \mathcal{C}^n\}$ such that $\mathcal{C}^i, \mathcal{C}^j \subset \mathcal{J}$ and $\bigcup_{i=1}^n \mathcal{C}_i = \mathcal{J}$ for $0 < i, j \leq n$. The coalition formation structure is called a *partition* of $\mathcal{J}$ if the coalitions are disjoint $\mathcal{C}^i \cap \mathcal{C}^j = \emptyset \ \forall i \neq j$ and $\bigcup_{i=1}^n \mathcal{C}_i = \mathcal{J}$.

The number of possible collections of coalitions of a grand coalition grows exponentially with the number of players. Therefore, finding a stable coalition formation structure is important. Let us define the preference notation in comparing different collections of coalitions as follows.

Definition 7: Assume $\mathcal{S} = \{\mathcal{S}^1, \mathcal{S}^2, \dots, \mathcal{S}^n\}$ and $\mathcal{T} = \{\mathcal{T}^1, \mathcal{T}^2, \dots, \mathcal{T}^m\}$ are two coalition formation structures of $\mathcal{J}$ with $\bigcup_{i\in\{1,2,\dots,n\}} \mathcal{S}^i = \bigcup_{j\in\{1,2,\dots,m\}} \mathcal{T}^j = \mathcal{J}$. Defining a comparison relation $\rhd$, $\mathcal{S} \rhd \mathcal{T}$ means that $\mathcal{S}$ is *preferable* to $\mathcal{T}$. In addition, let us define the *Pareto order* for the comparison relation as follows. $\mathcal{S} \rhd \mathcal{T}$ means $\varpi_{P_j}(\mathcal{S}) \geq \varpi_{P_j}(\mathcal{T}), \ \forall \ P_j \in \mathcal{S}, \mathcal{T}$ with at least one strict inequality ($>$) for a PU P_j where $\varpi_{P_j}(\mathcal{S})$ is the payoff of PU P_j in a coalition formation structure $\mathcal{S}$.

Definition 8: We say a collection of coalitions $\mathcal{S} = \{\mathcal{S}^1, \mathcal{S}^2, \dots, \mathcal{S}^n\}$ of $\mathcal{J}$ is *stable* if none of players has incentive to leave $\mathcal{S}$, i.e., for all collections $\mathcal{T} \neq \mathcal{S}$, $\mathcal{S} \rhd \mathcal{T}$ holds. In the case that $\rhd$ represents the Pareto order, we say the coalition formation structure $\mathcal{S}$ is a *Pareto optimal* payoff distribution for all the players.

We have the following results about the feasible region of the pricing coefficients. We assume that PUs can use common knowledge or previous observation about SU networks to estimate the approximate ranges of some parameters for SUs. Combining the power constraints in (2) with the payoff functions of PUs and SUs, we have the following bounds for the pricing coefficients.

Proposition 3: Suppose that $\underline{h} < h_{jk[l]} < \overline{h}, 0 < g_{S_k[l]} < \overline{g}, \ \forall \ P_j \in \mathcal{J}, S_k \in \mathcal{K}$. Then each PU P_j only needs to adjust its pricing coefficient $\beta_{P_j[l]}$ within the range of $0 < \beta_{P_j[l]} < \overline{\beta}$ $\forall P_j \in \mathcal{J}, l \in \mathcal{M}$ where

$$\overline{\beta} = \frac{1}{J\underline{h}} \frac{\overline{h}}{\min\limits_{P_j\in\mathcal{J}} \left\{\underline{q}_{P_j}\right\} + \frac{\overline{h}}{\overline{g}}}. \tag{16}$$

Proof: See Appendix E. ∎

The above proposition defines a feasible region of the pricing coefficient of P_j. In other words, each PU P_j only needs to search for the optimal pricing coefficient $\boldsymbol{\beta}^*_{P_j}$ within the region of $[0, \overline{\beta}]$.

In our model, each SU first estimates its payoffs in all the sub-bands as if it is the only SU in these sub-bands, and then waits for a short period of time before accessing the sub-bands. We denote the maximum waiting time of SUs as $\bar{\tau}$. Let us now describe the algorithm below.

Algorithm 1: A Joint Optimization Algorithm

Definitions and assumptions: At iteration t,

- Let $\mathcal{L}_{P_j}(t)$ be the set of sub-bands which are charged by PU P_j,
- Let $\mathcal{C}_{[l]}(t)$ be the set of PUs who need to charge SU in sub-band l,
- Let $\Delta\mathcal{C}_{[l]}(t)$ be the set of PUs who charge SU in sub-band l for the first time, i.e., $\mathcal{C}_{[l]}(t) = \mathcal{C}_{[l]}(t-1) \cup \Delta\mathcal{C}_{[l]}(t)$ and $\mathcal{C}_{[l]}(t-1) \cap \Delta\mathcal{C}_{[l]}(t) = \emptyset$,
- Let ϵ be the iteration step size which is a small constant satisfying $\epsilon \ll \overline{\beta}$ and $T = \frac{\overline{\beta}}{\epsilon}$ is an integer.
- Let $\mathcal{U}_{S_k}(t)$ be the set of sub-bands that S_k tries to access,
- $\bar{\tau} \ll \bar{D}$ is a constant where $\bar{D}$ is the time duration between the price changing of PUs and γ is a constant known by all SUs satisfying $\gamma \geq \bar{\tau} \max\limits_{S_k\in\mathcal{K}, l\in\mathcal{M}} \varpi_{S_k[l]}$.

1) *Initialization*:
   a) Set $\mathcal{C}_{[l]}(0) = \emptyset$ and $\mathcal{L}_{P_j}(0) = \emptyset$,
   b) Each PU P_j broadcasts the pricing coefficients $\boldsymbol{\beta}_{P_j}(0) = [\beta_{P_j[1]}(0), \ \beta_{P_j[2]}(0) \ , \dots, \ \beta_{P_j[M]}(0)]$ where $\beta_{P_j[l]}(0) = \overline{\beta} \ \forall l \in \mathcal{M}$.[4]
2) *Coalition Formation*: For $0 \leq t \leq T$,
   a) Receiving $\boldsymbol{\beta}(t)$, the SUs sequentially send a one-bit training message to estimate their payoff in all the sub-bands. SU S_k knows $\varpi_{S_k[1]}, \varpi_{S_k[2]}, \dots, \varpi_{S_k[M]}$ where $\varpi_{S_k[m]}$ is the payoff of SU S_k when sub-band m is exclusively used by SU S_k. Each SU $S_k \in \mathcal{K}$ waits a short time duration $\tau_{S_k[l]} = \frac{\gamma}{\varpi_{S_k[l]}} < \bar{\tau}$ before accessing sub-band l. If $t < \tau_{S_k[l]}$, each SU S_k updates the set $\mathcal{U}_{S_k}(t) = \mathcal{U}_{S_k}(t) \backslash \{l\}$ whenever it receives $l \in \mathcal{L}_{S_n}$ broadcast by an SU S_n for $\tau_{n[l]} < \tau_{k[l]}$. If SU S_k occupies sub-band l, it sets its transmit power $w^*_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}(t), \mathcal{C}_{[l]}(t)\right)$ and inform all PUs in $\mathcal{C}_{[l]}$ its value of $g_{S_k[l]}$.
   b) At iteration t, if a PU P_j cannot charge any SUs, i.e., $\mathcal{L}_{P_j} = \emptyset$, then go to Step 3) directly. If the PU P_j charges the transmission of SUs in at least one sub-band, it sends the list $\mathcal{L}_{P_j}(t)$ to other PUs for possible cooperation.
   c) If this is the first time for PU $P_j \in \Delta\mathcal{C}_{[l]}(t)$ to charge the transmission of an SU S_k in sub-band l, P_j will search for the previously received SU lists $\mathcal{C}_{[l]}(t-1)$,
      i) If $\mathcal{C}_{[l]}(t-1) = \emptyset$, a coalition $\mathcal{C}_{[l]}(t)$ will be formed to decide the price $u_{S_k[l]}$ of the SU S_k in sub-band l such that

$$\mathcal{C}_{[l]}(t) = \left\{P_j : h_{jk[l]} w^*_{S_k[l]}(t) \geq \underline{q}_{P_j}, \ \forall \ P_j \in \mathcal{J}\right\},$$

and all PUs $P_j \in \mathcal{C}_{[l]}(t)$ will jointly calculate the optimal sequential coalition $\mathcal{C}^*_{[l]}(t)$ as follows:

$$\mathcal{C}^*_{[l]}(t) = \arg\max_{\mathcal{C}_{[l]}\in\tilde{\mathcal{C}}_{[l]}} \sum_{P_j\in\mathcal{C}_{[l]}} \varpi_{P_j}\left(\boldsymbol{w}^*_S, \boldsymbol{\beta}^*_{[l]}, \mathcal{C}_{[l]}, \boldsymbol{\mathcal{L}}_S\right), \tag{17}$$

where $\boldsymbol{\beta}^*_{[l]}$ is calculated from $u^*_{S_k}\left(\boldsymbol{\beta}_{[l]}(t), \mathcal{C}_{[l]}\right)$ using (13). Go to Step 3),

[4]Note that, at the beginning of each iteration, PUs need to pre-set the prices for each frequency band of SUs without knowing how many SUs can afford the price. Hence, we abuse the notation and use $\beta_{P_j[l]}(t)$ to denote the price that PU P_j sets for use of sub-band l of SU S_k even if $w^*_{S_k[l]} = 0$.

ii) If $\mathcal{C}_{[l]}(t-1) \neq \emptyset$, PU $P_j \in \Delta\mathcal{C}_{[l]}(t)$ will negotiate with PU $P_i \in \mathcal{C}_{[l]}(t-1)$ for a possible new division of revenue from SU S_k. Then, all PUs $P_j \in \mathcal{C}_{[l]}(t)$ will update $\mathcal{C}_{[l]}(t) = \mathcal{C}_{[l]}(t-1) \cup \Delta\mathcal{C}_{[l]}(t)$ and then use (17) to calculate the optimal sequential coalition. Go to Step 3),

d) If a PU P_j has already joined the coalition to decide the price charging to SU S_k in the previous iteration, i.e., $P_j \in \mathcal{C}_{[l]}(t-1)$,

i) If $\Delta\mathcal{C}_{[l]}(t) \neq \emptyset$, PU P_j updates $\mathcal{C}_{[l]}(t) = \mathcal{C}_{[l]}(t-1) \cup \Delta\mathcal{C}_{[l]}(t)$ and then uses (17) to calculate the optimal sequential coalition. Go to Step 3),

ii) If $\Delta\mathcal{C}_{[l]}(t) = \emptyset$, directly go to Step 3).

3) *Dynamic Coalition Updating*: At the end of iteration t,

a) If $\mathcal{L}_{P_j}(t) = \emptyset$, PU P_j will update the price $\boldsymbol{\beta}_{P_j}(t) = \boldsymbol{\beta}_{P_j}(t-1) - \epsilon$ for all frequency bands.

b) If $\mathcal{L}_{P_j}(t) \neq \emptyset$, PU P_j will jointly collaborate with other PUs $P_j, P_i \in \mathcal{C}_{[l]}$ $\forall l \in \mathcal{L}_{P_j}$ to update the price

$$u_{S_k}\left(\boldsymbol{\beta}_{[l]}(t+1), \mathcal{C}_{[l]}(t+1)\right) = u_{S_k}\left(\boldsymbol{\beta}_{[l]}(t) - \epsilon, \mathcal{C}_{[l]}(t)\right)$$

for $l \in \mathcal{L}_{P_j}$. In addition, PU P_j will also update the price

$$\beta_{P_j[l]}(t) = \beta_{P_j[l]}(t-1) - \epsilon, \ \ \forall l \notin \mathcal{L}_{P_j}(t), l \in \mathcal{M}$$

for the rest of sub-bands.

Let $t = t+1$. Go to Step 2).

4) *Termination*: If one PU $P_j \in \mathcal{C}_{[l]}$ detects higher than tolerable interference from SU S_k, i.e., $h_{jk[l]} w^*_{S_k[l]} \geq \overline{q}_{P_j}$, it will broadcast a “stop” message to all the members in coalition $\mathcal{C}_{[l]}$, and then all the PUs in coalition $\mathcal{C}_{[l]}$ will stop decreasing $\beta_{P_j[l]}$ $\forall P_j \in \mathcal{C}_{[l]}$.

a) If $h_{jk[l]} w^*_{S_k[l]}(t) > \overline{q}_{P_j}$, $\forall$ $S_k \in \mathcal{K}$ and $t \leq T$, the algorithm ends with solution $\mathcal{C}^*_{[l]}(t^*_{[l]})$ and $u_{S_k}\left(\boldsymbol{\beta}_{[l]}(t^*_{[l]}), \mathcal{C}^*_{[l]}(t^*_{[l]})\right)$ $\forall S_k \in \mathcal{K}, P_j \in \mathcal{J}, l \in \mathcal{M}$ where $t^*_{[l]}$ is given by

$$t^*_{[l]} = \arg \max_{t \in \{0,1,\ldots,T\}} \sum_{P_j \in \mathcal{C}^*_{[l]}(t)} \varpi_{P_j}\left(\boldsymbol{w}^*_S(t), \boldsymbol{\beta}_{[l]}(t), \mathcal{C}^*_{[l]}(t), \boldsymbol{\mathcal{L}}_S(t)\right), \quad (18)$$

and $\mathcal{C}^*_{[l]}(t)$ is given by (17).

b) Else the algorithm ends when $t = T$.

Note that in the above algorithm, the effects of the cooperation cost has only been evaluated in (17) and (18). This is because, in some systems, the total payoff of a coalition may increase with more PUs to join, e.g., in systems shown effects of positive network externalities [33]. Since the maximum number of sequential subsets of a set $\mathcal{C}_{[l]}$ is at most $|\mathcal{C}_{[l]}|$, the maximization operations in (17) and (18) only require less than $|\widetilde{\mathcal{C}}_{[l]}(t)|$ and T numbers of searches, respectively.

The following theorem shows that Algorithm 1 achieves both a unique, stable and $\triangleright$ maximal collection of coalitions for the coalition formation game, as well as a pure strategy SE for the hierarchical game defined in Proposition 1.

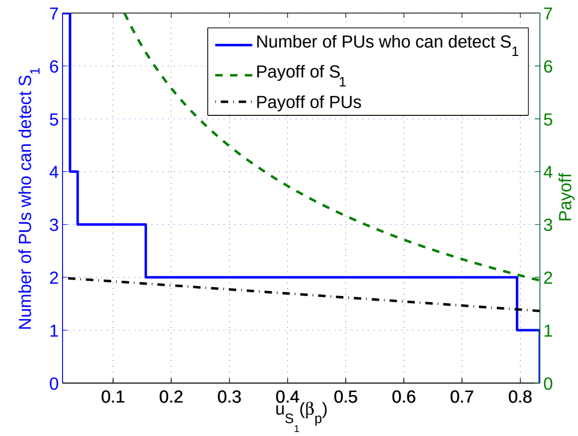

Fig. 4. Coalition formation under different $u_{S_1}(\boldsymbol{\beta}_P)$.

Theorem 1: If Algorithm 1 terminates, either we have $\mathcal{C}_{[l]} = \emptyset$, for $l \in \mathcal{M}$, or we have

1) The sub-band allocation scheme $\boldsymbol{\mathcal{L}}_S$ is an NE of the sub-band allocation game given the resulting $\boldsymbol{\beta}^*$;
2) If (13) is satisfied, the collection of coalition is unique, stable and $\triangleright$ maximal for a sequential coalitional game with the resulting $\boldsymbol{\mathcal{L}}_S$, and $(\boldsymbol{w}^*_S, \boldsymbol{\beta}^*)$ is a pure strategy SE for the hierarchical game with the given $\boldsymbol{\mathcal{L}}_S$;
3) Otherwise $(\boldsymbol{w}^*_S, \boldsymbol{\beta}^*)$ is within an ϵ distance of an SE for the hierarchical game with the resulting $\boldsymbol{\mathcal{L}}_S$.

Proof: See Appendix F. ■

Note that the results in 2) and 3) of Theorem 2 also hold when SUs use any other sub-band allocation scheme in Algorithm 1. This is because for any resulting sub-band allocation scheme, using Algorithm 1, PUs can always find the lowest pricing coefficients to make at least one PU observe a higher than tolerable interference.

It is observed that the maximum number of iterations required for Algorithm 1 is given by $T \leq \frac{\overline{\beta}}{\epsilon}$, which is independent of the number of SUs or PUs where $\overline{\beta}$ is a constant defined in Proposition 3.

In Figure 4, we show the size of a coalition $\mathcal{C}_{[l]}$ in sub-band l and the payoff of S_k under different values of $u_{S_k}\left(\boldsymbol{\beta}_{[l]}, \mathcal{C}_{[l]}\right)$. It is observed that the size of the coalition as well as the payoffs of SUs decrease with $u_{S_k}\left(\boldsymbol{\beta}_{[l]}, \mathcal{C}_{[l]}\right)$. This verifies our previous observations that PUs can use $\boldsymbol{\beta}$ to control the collections of the grand coalition as well as the payoffs of SUs and PUs.

VI. Fairness Criteria for Payoff Division within Each Coalition

Algorithm 1 presents a distributed coalition formation solution. However, it does not describe how to fairly divide the payoff among the members in each coalition. In this section, we focus on one coalition $\mathcal{C}_{[l]}$ in sub-band l and investigate different fairness criteria for dividing the payoff obtained from one SU S_k among the PUs. We can then drop the term $(\mathcal{C}_{[l]})$ and write the cooperation cost of each PU P_j in $\mathcal{C}_{[l]}$ as $\theta_{P_j[l]}$ without causing any confusion. In this section, we always assume that the core of the coalitional game $\left(\mathcal{C}_{[l]}, v\right)$ is nonempty. In the rest of this section, we study the payoff allocations among PUs in each coalition using two of the most widely studied fairness criteria, i.e., Nash bargaining solution and Shapley value, both of which belong to the axiomatic approach in the game theory [34].

A. Nash Bargaining Solution Fairness

Nash bargaining solution characterizes the outcome of a bargaining process among PUs who can jointly decide the prices of the same sub-bands. It is closely related to the proportional fairness which has already been widely applied into mobile networks [35], [36].

Let $\mathcal{F}$ be a closed convex subset of $\mathbb{R}^J$ that represents the set of feasible payoff allocations of PUs in one coalition $\mathcal{C}_{[l]}$. Let $\varpi_{P_j}^{\min}$ be the disagreement payoff allocation of PU P_j, i.e., PU P_j will not cooperate with other PUs if $\varpi_{P_j[l]} < \varpi_{P_j[l]}^{\min}$. Suppose $\left\{\varpi_{P_j[l]} \in \mathcal{F} | \varpi_{P_j[l]} \geq \varpi_{P_j[l]}^{\min}\right\}$ is nonempty. We define a J-player Nash bargaining problem to consist of a pair $\left(\mathcal{F}, \boldsymbol{\varpi}_{[l]}^{\min}\right)$ where $\boldsymbol{\varpi}_{[l]}^{\min} = \left(\varpi_{P_j[l]}^{\min}\right)_{P_j \in \mathcal{C}_{[l]}}$.

Definition 9: [13, Chapter 8] $\boldsymbol{\varpi}_{[l]}^*$ is said to be a *Nash bargaining solution* in $\mathcal{F}$ with $\boldsymbol{\varpi}_{[l]}^{\min}$ for the coalition $\mathcal{C}_{[l]}$, i.e., $\boldsymbol{\varpi}_{[l]}^* = \phi\left(\mathcal{F}, \boldsymbol{\varpi}_{[l]}^{\min}\right)$ if the axioms of *feasibility*, *efficiency*, *individual rationality*, *scale covariance*, *independence of irrelevant alternatives*, *symmetry*[5] are satisfied.

Proposition 4: [13, Theorem 8.1] There is a unique solution function $\phi(\cdot,\cdot)$ that satisfies all five axioms in Definition 9. This solution satisfies

$$\phi\left(\mathcal{F}, \boldsymbol{\varpi}^{\min}\right) \in \arg \max_{\substack{\boldsymbol{\varpi}_{[l]}^* \in \mathcal{F}, \\ \boldsymbol{\varpi}_{[l]}^* \succeq \boldsymbol{\varpi}_{[l]}^{\min}}} \prod_{P_j \in \mathcal{C}_{[l]}} \left(\varpi_{P_j[l]}^* - \varpi_{P_j[l]}^{\min}\right) \quad (19)$$

where $\boldsymbol{\varpi}_{[l]}^* \succeq \boldsymbol{\varpi}_{[l]}^{\min}$ denotes $\varpi_{P_j[l]}^* \geq \varpi_{P_j[l]}^{\min}$ for all $P_j \in \mathcal{C}_{[l]}$.

Theorem 2: There exists a unique Nash bargaining solution for a coalition $\mathcal{C}_{[l]}$ in the proposed game, which is given by $\boldsymbol{\beta}_{P_j} = \left(\beta_{P_j[l]}\right)_{P_j \in \mathcal{C}_{[l]}}$ where $\beta_{P_j[l]}$ is given by,

$$\beta_{P_j[l]} = \frac{1}{J h_{jk[l]}} \left(\frac{1}{\zeta + \frac{1}{g_{S_k[l]}}} + \frac{J \hat{\theta}_{P_j[l]} - \hat{\theta}_{\mathcal{C}_{[l]}}}{\zeta} \right) \quad (20)$$

where $\zeta = \min_{P_j \in \mathcal{C}_{[l]}} \left\{ \frac{\overline{q}_{P_j}}{h_{jk[l]}} \right\}$, $\hat{\theta}_{P_j[l]} = \theta_{P_j[l]} + \varpi_{P_j[l]}^{\min}$ and $\hat{\theta}_{\mathcal{C}_{[l]}} = \sum_{P_j \in \mathcal{C}_{[l]}} \hat{\theta}_{P_j[l]}$.

Proof: See Appendix G. ∎

The main problem with the Nash bargaining solution is that it neglects the dynamics of the coalition formation process. More specifically, during the price updating process in Algorithm 1, some PUs (i.e., the PUs who are close to the SUs) can always join a coalition earlier than the other PUs. In this case, if these earlier joiners cannot obtain higher payoffs than that of the later ones, they will lose the incentive to further reduce prices to allow more PUs to join the coalition. Thus, in the rest of this section, we consider a fairness criterion that can take the contribution of each PU during the coalition formation process into account.

B. Shapley Value Fairness

Let us consider the fairness of each PU using the Shapley value.

Definition 10: [13, Chapter 9.4] Let $\mathcal{L}(\mathcal{C})$ be the set of all possible coalitions among the PUs in the coalition $\mathcal{C}$. A *Shapley value* is a mapping $\phi : \mathbb{R}^{|\mathcal{L}(\mathcal{C})|} \to \mathbb{R}^{|\mathcal{C}|}$ such that, when the PUs in $\mathcal{C}$ play any coalitional game v, the expected payoff to each player i would be $\phi_{P_i}(v)$, i.e., $\boldsymbol{\phi}(v) = \left(\phi_{P_1}(v), \phi_{P_2}(v), \ldots, \phi_{P_{|\mathcal{C}|}}(v)\right)$. A Shapley value $\boldsymbol{\phi}$ must satisfy the axioms of *symmetry*, *efficiency*, *dummy* and *additive*[6].

It is proved in [13] that there exists a unique function $\boldsymbol{\phi}$ satisfying all the above axioms given by

$$\phi_{P_j}(v) = \sum_{\mathcal{S} \subseteq \mathcal{C} \setminus \{P_j\}} \frac{(|\mathcal{S}|)!\,(|\mathcal{J}| - |\mathcal{S}| - 1)!}{(|\mathcal{J}|)!} \left[v\left(\mathcal{S} \cup \{P_j\}\right) - v\left(\mathcal{S}\right) \right] \quad (21)$$

Theorem 3: The Shapley value of a coalition $\mathcal{C}_{[l]}$ in the proposed game is always in the core and the value of $\boldsymbol{\beta}$ that yields the Shapley value fairness of a coalition $\mathcal{C}[l]$ is given by

$$\beta_{P_{\tilde{j}}[l]} = \begin{cases} \left(\frac{\eta_{P_{\tilde{j}}[l]} - \eta_{P_{\widetilde{j-1}}[l]} - \Delta\theta_{P_{\tilde{j}}[l]}}{h_{\tilde{j}k[l]}\zeta} \right), & \text{if } 1 \leq j \leq J-1, \\ \left(\frac{\eta_{P_J[l]} - \eta_{P_{\widetilde{J-1}}[l]} - \Delta\theta_{P_{\tilde{J}}[l]}}{h_{\tilde{j}k[l]}\zeta} \right), & \text{if } j = J, \end{cases} \quad (22)$$

where ζ is defined after (20), $\eta_{P_{\tilde{0}}[l]} = 0$, $\eta_{P_{\tilde{j}}[l]} = \frac{g_{S_k[l]} \underline{q}_{P_{\tilde{j}}[l]}}{\underline{q}_{P_{\tilde{j}}} g_{S_k[l]} + h_{\tilde{j}k[l]}}$, $\eta_{P_J[l]} = \frac{g_{S_k[l]}\zeta}{g_{S_k[l]}\zeta + 1}$. and $\Delta\theta_{P_{\tilde{j}}[l]} = \sum_{\tilde{i} \in \{\tilde{1}, \tilde{2}, \ldots, \tilde{j}\}} \theta_{P_{\tilde{i}}[l]}\left(\left\{\tilde{1}, \tilde{2}, \ldots, \tilde{j}\right\}\right) - \sum_{\tilde{i} \in \{\tilde{1}, \tilde{2}, \ldots, \widetilde{j-1}\}} \theta_{P_{\tilde{i}}[l]}\left(\left\{\tilde{1}, \tilde{2}, \ldots, \widetilde{j-1}\right\}\right)$.

Proof: See Appendix H. ∎

VII. Numerical Results

To evaluate the performance of our algorithm, we consider a panel network in which two PUs, P_1 and P_2 are located at the center, and all the SUs are randomly located in the networks as shown in Figure 5. We assume that each SU can only access one sub-band and hence can use S_k to denote the corresponding sub-band of SU S_k. Let the pricing coefficients, charging threshold and interference limit of both PUs be fixed. We compare the performance of both SU and PU networks under different numbers of active SUs in Figures 6 - 8. It is observed in Figure 6 that, although the number of SUs being detected by PUs increases with the number of overall SUs, there always exist some SUs that can only be detected by

[5]We omit the details of these axioms due to space limit. Interest readers please refer to pages 377-378 in [13].

[6]We omit the details of these axioms due to space limit. Please see pages 437 - 438 in [13] for the details.

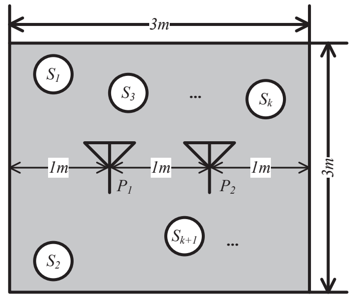

Fig. 5. Pricing coalition formation setup in a panel network.

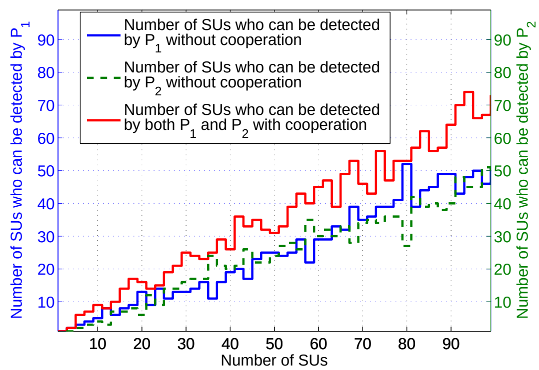

Fig. 6. The size of the pricing coalition of PUs in different numbers of SUs. Note that the fluctuation of the number of charged SUs is because we randomly generate the locations of all the SUs whenever the number of SUs changes.

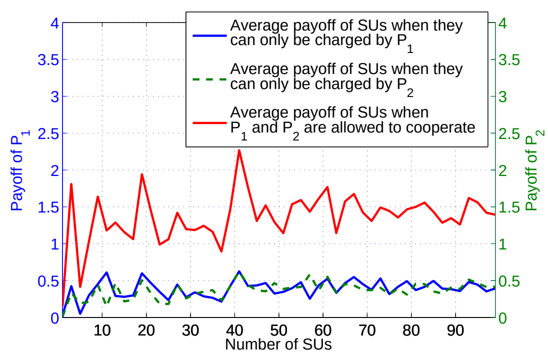

Fig. 7. Average payoff of SUs for different numbers of SUs.

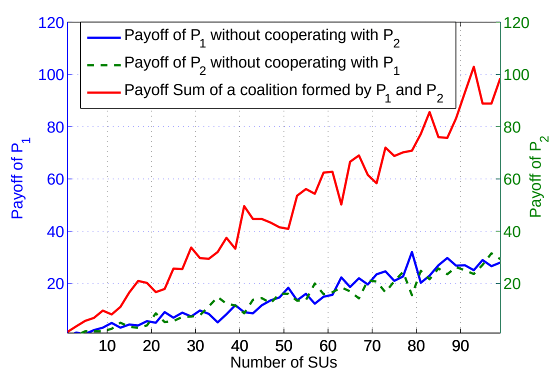

Fig. 8. Payoffs of PUs for different numbers of SUs.

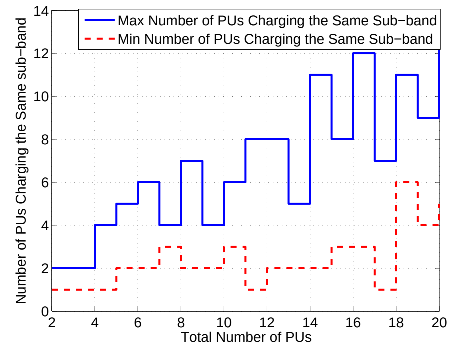

Fig. 9. The maximum and minimum numbers of PUs charging the same sub-band for different numbers of PUs.

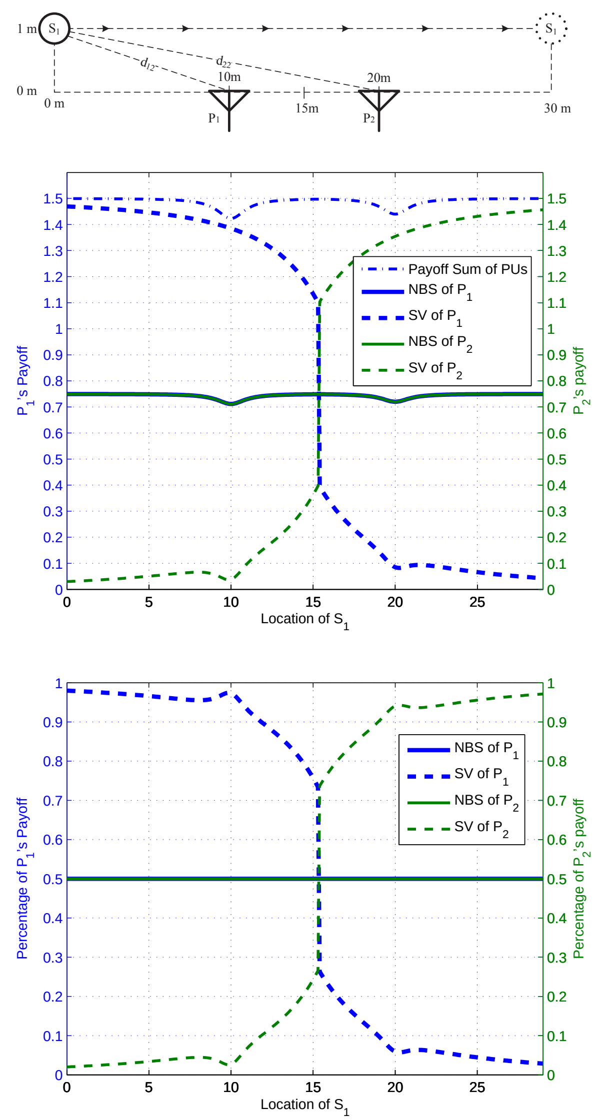

Fig. 10. The payoff divisions between P_1 and P_2 on two fairness criteria: Nash bargaining solution (NBS) fairness and Shapley value (SV) fairness.

one PU. In Figures 7 and 8, we observe that the coalition formation between PUs using Algorithm 1 always increases the payoffs of SUs and PUs. This verifies our conclusions in Observations 1 and 2 and Remark 1 in Section IV-B. In Figure 9, we consider the number of PUs charging the same sub-band of SUs under different numbers of PUs. We observe that the coalitions of PUs can be highly overlapped. More specifically, in may cases, every sub-band is charged by more than one PU (i.e., the minimum number of PUs charging the same sub-band exceeds two) and there always exists at least one sub-band that is charged by more than half of the PUs.

To compare the performance of different fairness criteria, we simulate a CR network in which two PUs, P_1 and P_2, try to divide the payoff obtained from one SU S_1 in a sub-band l. Both PUs have the same threshold and interference limit. The pricing coefficients $\beta_{P_1[l]}$ and $\beta_{P_2[l]}$ are fixed. Assume that channel gain h_{j1} is given by $h_{j1} = \frac{\hat{h}_{j1}}{d^{\xi}}$ for $j \in \{1,2\}$ where $\hat{h}_{j1}$ is the average channel path loss coefficient, d_{11} and d_{21} are the distances between S_1 and P_1, and S_1 and P_2, respectively. ξ is the fading exponent. Consider the case of both PUs located in a linear network and SU S_1 moving from the left to the right as shown at the top of Figure 10. Assume that SU S_1 is detectable to both PUs during its moving.

In the middle and bottom of Figure 10, we show the payoffs and the percentages of payoff for PUs P_1 and P_2 under different fairness criteria. It is observed that the payoff sum of PUs decreases when SU S_1 moves closer to either PU. This is because, as shown in Appendix A, the payoff sum of PUs decreases with the transmit power of SU S_1, which is minimized when the distance between SU S_1 and either PU becomes the shortest. Figure 10 also compares the payoff division between two PUs under different fairness criteria. More specifically, the NBS equally divides the payoff among PUs while the Shapley value divide the payoff sum according to the contribution brought by each member PU. It is observed that the Shapley value fairness criterion distributes most of the payoff to one PU P_j when S_1 becomes close to PU P_j for $j \in \{1,2\}$. This is because in our simulation, when SU S_1 has the shortest distance to either PU, the transmit power of S_1 will be minimized to the value that almost cannot be charged by P_{-j}, and hence the possible contribution of P_{-j} to join the coalition approaches zero.

VIII. Conclusion

In this paper, we have presented a hierarchical model for CR networks to study the interaction between SUs and PUs in a spectrum pooling system. We have proved that allowing all PUs to compete with each other without exchanging any information is generally not optimal. We have then proposed a pricing coalitional game framework to investigate the possible pricing coalition among PUs. We have observed that the grand coalition of the pricing coalitional game is generally not stable, and hence a simple algorithm has been proposed to allow PUs to distributedly form a unique and stable collection of coalitions. To further inspect the fairness payoff division problem for PUs within each coalition, two fairness criteria, Nash bargaining solution and Shapley value fairness, have been studied and compared.

Acknowledgment

The authors would like to thank Professor Luiz A. DaSilva for his helpful comments to the initial version of this paper.

Appendix

A. Proof of Proposition 1

Let us assume that the sub-band allocation schemes $\boldsymbol{\mathcal{L}}_S$ and $\boldsymbol{\mathcal{C}}$ are fixed and $l \in \mathcal{L}_{S_k}$. It can be shown that $\boldsymbol{\beta}$ takes values in a nonempty compact set and the payoff function $\varpi_{\mathcal{J}}\left(\boldsymbol{w}_S^*, \boldsymbol{\beta}, \boldsymbol{\mathcal{L}}_S, \boldsymbol{\mathcal{C}}\right)$ is continuous in this set. In addition, if $w_{S_k[l]} > \min_{P_j \in \mathcal{J}} \left\{ \frac{q_{P_j}}{h_{jk[l]}} \right\}$, for a given $\boldsymbol{\beta}_{[l]}$, $\varpi_{S_k[l]}\left(w_{S_k[l]}, \boldsymbol{\beta}_{[l]}, \boldsymbol{\mathcal{L}}_S\right)$ is a quasi-concave function of $w_{S_k[l]}$ with the optimal solution $w^*_{S_k[l]}$ given in (12). By substituting $w^*_{S_k[l]}$ into $\varpi_{S_k[l]}\left(w_{S_k[l]}, \boldsymbol{\beta}_{[l]}, \boldsymbol{\mathcal{L}}_S\right)$, we have

$$\begin{aligned}
&\varpi_{S_k[l]}\left(w^*_{S_k[l]}, \boldsymbol{\beta}_{[l]}, \boldsymbol{\mathcal{L}}_S\right) \\
&\quad = -\log_2\left(1 + \left(\frac{g_{S_k[l]}}{u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}, \mathcal{C}_{[l]}\right)} - 1\right)^+\right) \\
&\qquad -\left(1 - \frac{u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}, \mathcal{C}_{[l]}\right)}{g_{S_k[l]}}\right)^+ . \qquad (23)
\end{aligned}$$

The above result shows that $\varpi_{S_k[l]}\left(w^*_{S_k[l]}, \boldsymbol{\beta}_{[l]}, \boldsymbol{\mathcal{L}}_S\right)$ decreases with $u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}, \mathcal{C}_{[l]}\right)$ if $w^*_{S_k[l]} > \min_{P_j \in \mathcal{J}} \left\{ \frac{q_{P_j}}{h_{jk[l]}} \right\}$.

Substituting $w^*_{S_k[l]}$ into $\varpi_{\mathcal{J}}\left(\boldsymbol{w}_S^*, \boldsymbol{\beta}, \boldsymbol{\mathcal{L}}_S, \boldsymbol{\mathcal{C}}\right)$, we have

$$\begin{aligned}
&\varpi_{\mathcal{J}}\left(\boldsymbol{w}_S^*, \boldsymbol{\beta}, \boldsymbol{\mathcal{L}}_S, \boldsymbol{\mathcal{C}}\right) = \sum_{P_j \in \mathcal{J}} \varpi_{P_j}\left(\boldsymbol{w}_S^*, \boldsymbol{\beta}, \boldsymbol{\mathcal{L}}_S, \boldsymbol{\mathcal{C}}\right) \\
&= \sum_{P_j \in \mathcal{J}} \sum_{l \in \mathcal{L}_{P_j}} \left(\left(\frac{h_{jk[l]}\beta_{P_j[l]}}{u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}, \mathcal{C}_{[l]}\right)} - \frac{h_{jk[l]}\beta_{P_j[l]}}{g_{S_k[l]}}\right)^+ \right. \\
&\qquad \left. -\theta_{P_j[l]}\left(\mathcal{C}_{[l]}\right)\right) \\
&= \sum_{l \in \mathcal{M}} \sum_{P_j \in \mathcal{C}_{[l]}} \left(\left(\frac{h_{jk[l]}\beta_{P_j[l]}}{u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}, \mathcal{C}_{[l]}\right)} - \frac{h_{jk[l]}\beta_{P_j[l]}}{g_{S_k[l]}}\right)^+ \right. \\
&\qquad \left. -\theta_{P_j[l]}\left(\mathcal{C}_{[l]}\right)\right) \\
&= \sum_{l \in \mathcal{M}} \left(\left(1 - \frac{u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}, \mathcal{C}_{[l]}\right)}{g_{S_k[l]}}\right)^+ - \theta_{\mathcal{C}_{[l]}}\left(\mathcal{C}_{[l]}\right)\right),
\end{aligned}$$

where $\theta_{\mathcal{C}_{[l]}}\left(\mathcal{C}_{[l]}\right) = \sum_{P_j \in \mathcal{C}_{[l]}} \theta_{P_j[l]}\left(\mathcal{C}_{[l]}\right)$ is a constant.

It is observed that $\varpi_{\mathcal{J}}\left(\boldsymbol{w}_S^*, \boldsymbol{\beta}\right)$ is a linearly decreasing function of $u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}, \mathcal{C}_{[l]}\right)$. We hence can claim that $\varpi_{S_k}\left(w^*_{S_k}, \boldsymbol{\beta}_{[l]}\right)$ and $\varpi_{\mathcal{J}}$ are maximized when $u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}, \mathcal{C}_{[l]}\right)$ is minimized in every sub-band. Combining (2) and (12), we can derive the lower bound of $u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}, \mathcal{C}_{[l]}\right)$ in (13).

B. Proof of Observation 1

Consider a CR network with two PUs, labeled as P_1 and P_2, and one SU, labeled as S_1 in sub-band l. Let us first focus on the case that both PUs can detect the existence of S_1. In this case, the following condition must be satisfied,

$$\max\left\{\frac{q_{P_1}}{h_{11[l]}}, \frac{q_{P_2}}{h_{12[l]}}\right\} < w_{S_1}\left(\beta_{P_1[l]}, \beta_{P_2[l]}\right) \tag{24}$$

$$= \left(\frac{1}{h_{11[l]}\beta_{P_1[l]} + h_{12[l]}\beta_{P_2[l]}} - \frac{1}{g_{S_1[l]}}\right)^+$$

$$< \min\left\{\frac{\overline{q}_{P_1}}{h_{11[l]}}, \frac{\overline{q}_{P_2[l]}}{h_{12[l]}}\right\}, \tag{25}$$

and the payoff of each PU P_j for $j \in \{1,2\}$ is given by $\varpi_{P_j[l]} = \beta_{P_i[l]} h_{1j[l]} w_{S_1[l]}\left(\beta_{P_1[l]}, \beta_{P_2[l]}\right)$. Note that P_j cannot know the exact value of $\beta_{P_{-j}[l]}$. If P_1 assumes that P_2 chooses the lowest possible pricing coefficient $\beta_{P_2[l]}$, then choosing the lowest $\beta_{P_1[l]}$ for P_1 will cause $w_{S_1}\left(\beta_{P_1[l]}, \beta_{P_2[l]}\right)$ to be equal to the maximum value given in the right-hand-side of (24), which is a constant unrelated to the pricing coefficients of both PUs. In this case, we can observe that the payoff of P_1, given by $\varpi_{P_1}(\beta_{P_1[l]}) = \beta_{P_1[l]} h_{11[l]} \min\left\{\frac{\overline{q}_{P_1[l]}}{h_{11[l]}}, \frac{\overline{q}_{P_2[l]}}{h_{12[l]}}\right\}$, increases with $\beta_{P_1[l]}$. Therefore, P_1 should increase its pricing coefficient $\beta_{P_1[l]}$ to the maximum possible value which will lead to the left-hand-side of (24). On the other hand, if P_1 assumes that P_2 chooses the highest possible $\beta_{P_2[l]}$, then $w_{S_1[l]}\left(\beta_{P_1[l]}, \beta_{P_2[l]}\right)$ will again result in the left-hand-side of (24) and hence following the same reason, P_1 will also choose the maximum value of $\beta_{P_1[l]}$. In other words, as long as one PU detects that an SU has been charged by multiple PUs in a sub-band, this PU will always choose the highest pricing coefficient for this SU. This will eventually cause the payoff of PU P_j to be zero for $P_j = \arg\max_{i\in\{1,2\}}\left\{\frac{q_{P_i[l]}}{h_{1i[l]}}\right\}$.

Let us now consider the case that the SU S_1 can only be charged by one PU P_1. In this case, we have

$$\frac{q_{P_1[l]}}{h_{11[l]}} < w_{S_1[l]}\left(\beta_{P_1[l]}\right) = \left(\frac{1}{h_{11[l]}\beta_{P_1[l]}} - \frac{1}{g_{S_1[l]}}\right)^+$$

$$< \min\left\{\frac{\overline{q}_{P_1[l]}}{h_{11[l]}}, \frac{q_{P_2[l]}}{h_{12[l]}}\right\}. \tag{26}$$

The payoff of P_1 is given by $\varpi_{P_1[l]} = \beta_{P_1[l]} h_{11[l]} w_{S_1[l]}\left(\beta_{P_1[l]}\right) = \left(1 - \frac{h_{11[l]}\beta_{P_1[l]}}{g_{S_1}}\right)^+$, which decreases with $\beta_{P_1[l]}$. In other words, if the SU can only be charged by one PU P_1, this PU will always choose the lowest pricing coefficient to maximize its payoff, which will result in a payoff sum of

$$\begin{aligned}\varpi_{P_1[l]} + \varpi_{P_2[l]} &= \beta_{P_1[l]} h_{11[l]} w_{S_1[l]}\left(\beta_{P_1[l]}\right) + 0 \\ &= \frac{\min\left\{\frac{\overline{q}_{P_1[l]}}{h_{11[l]}}, \frac{q_{P_2[l]}}{h_{12[l]}}\right\}}{\min\left\{\frac{\overline{q}_{P_1[l]}}{h_{11[l]}}, \frac{q_{P_2[l]}}{h_{12[l]}}\right\} + \frac{1}{g_{S_1[l]}}}.\end{aligned} \tag{27}$$

However, as shown in Proposition 1 and Appendix A that if both PUs can cooperate to decide the price of S_1 and $\sum_{j=\{1,2\}} \theta_{P_j[l]}\left(\{P_1, P_2\}\right) \to 0$, the optimal payoff sum of both PUs should be given by

$$\varpi_{P_1[l]} + \varpi_{P_2[l]} = \frac{\min\left\{\frac{\overline{q}_{P_1[l]}}{h_{11[l]}}, \frac{\overline{q}_{P_2[l]}}{h_{12[l]}}\right\}}{\min\left\{\frac{\overline{q}_{P_1[l]}}{h_{11[l]}}, \frac{\overline{q}_{P_2[l]}}{h_{12[l]}}\right\} + \frac{1}{g_{S_1[l]}}}. \tag{28}$$

We hence can claim that (27) is always less than (28) when $\underline{q}_{P_j} < \overline{q}_{P_{-j}}$ for $j \in \{1,2\}$. The above observation can be directly extended into the CR networks with J PUs and K SUs. We omit the detailed discussion due to space limit.

C. Proof of Observation 2

In the proposed model, if PU P_j cannot charge an SU S_k that uses sub-band l, the PU cannot obtain any revenue from sub-band l, i.e., $R_{P_j[l]} = 0$. Combining this observation with $\varpi_{P_j[l]} = R_{P_j[l]} - \theta_{P_j[l]}(\mathcal{J}) < 0$, we have

$$\begin{aligned}v(\mathcal{J}\times\mathcal{M}) &= \sum_{P_i\times l\in\mathcal{J}\times\mathcal{M}} \varpi_{P_i[l]} \\ &= \sum_{P_i\times l\in\mathcal{J}\times\mathcal{M}\backslash\{P_j\}\times\{l\}} \left(\varpi_{P_i[l]} + \varpi_{P_j[l]}\right) \\ &< \sum_{P_i\times l\in\mathcal{J}\times\mathcal{M}\backslash\{P_j\}\times\{l\}} \varpi_{P_i[l]} \\ &= v(\mathcal{J}\times\mathcal{M}\backslash\{P_j\}\times\{l\}).\end{aligned}$$

$\times$ denotes a Cartesian product operation.

This means that if PU P_j joins the grand coalition in sub-band l, it cannot provide a positive payoff improvement. However, if P_j does not cooperate with any PUs in sub-band l, its payoff is zero. In other words, it is impossible to find an imputation that lies in the core of the grand coalition defined in (14). This concludes our proof.

D. Proof of Proposition 2

To prove Proposition 2, we only need to show that any coalition $\mathcal{C}_{[l]} = \mathcal{C}^1_{[l]} \cup \mathcal{C}^2_{[l]}$ formed by combining two sequential coalitions $\mathcal{C}^1_{[l]}$ and $\mathcal{C}^2_{[l]}$ improves the total revenue, i.e., $R_{\mathcal{C}_{[l]}} > R_{\mathcal{C}^1_{[l]}} + R_{\mathcal{C}^2_{[l]}}$. We then can use condition 3) to verify the super-additive condition. More specifically, assume $P_{\widetilde{1}} \in \mathcal{C}^1_{[l]}$ and $\boldsymbol{\beta}_{[l]}$ is fixed. We have

$$\begin{aligned}R_{\mathcal{C}^1_{[l]}} + R_{\mathcal{C}^2_{[l]}} &= \sum_{P_j\in\mathcal{C}^1_{[l]}} \left(\beta_{P_j[l]} h_{jk} \frac{q_{P_{\widetilde{M}}}}{h_{\widetilde{M}k}}\right) \\ &< R_{\mathcal{C}_{[l]}} = \sum_{P_j\in\mathcal{C}_{[l]}} \left(\beta_{P_j[l]} h_{jk} \frac{q_{P_{\widetilde{N}}}}{h_{\widetilde{N}k[l]}}\right)\end{aligned} \tag{29}$$

where $M = |\mathcal{C}^1_{[l]}| + 1$ and $N = |\mathcal{C}^1_{[l]} \cup \mathcal{C}^2_{[l]}| + 1$. Note that in (29), the revenue of $R_{\mathcal{C}^2_{[l]}}$ equals zero if $\mathcal{C}^2_{[l]}$ cannot cooperate with $\mathcal{C}^1_{[l]}$. This completes our proof.

E. Proof of Proposition 3

Let us now prove that it is not necessary for each PU to consider a value of pricing coefficient that is larger than $\overline{\beta}$. Suppose that every PU P_j chooses a value $\beta_{P_j[l]} > \overline{\beta} > 0 \; \forall P_j \in \mathcal{J}$. Then substituting (16) into $w^*_{S_k}(\boldsymbol{\beta}_{[l]})$ and using some operations, we have

$$w^*_{S_k[l]}(\boldsymbol{\beta}_{[l]}) < \frac{1}{\sum\limits_{P_j \in \mathcal{J}} h_{jk[l]}\overline{\beta}} - \frac{1}{g_{S_k[l]}} < \min_{P_j \in \mathcal{J}} \left\{ \frac{q_{P_j}}{h_{jk[l]}} \right\}, \quad (30)$$

which means that if $\beta_{P_j[l]} > \overline{\beta}$, no PUs can detect the presence of the SU S_k. According to our setup, this means that no PU can obtain any payoff from the licensed spectrum. The proof is now complete.

F. Proof of Theorem 1

Let us consider first result in Theorem 1. It can be easily shown that by following the sub-band allocation scheme in Step 2-a) of Algorithm 1, the waiting time of each SU to join sub-band l decreases with its payoff and hence the SU achieving the highest payoff in a sub-band can always occupy the sub-band earlier than other SUs. In addition, as observed in Appendix A, the payoff of each SU S_k is only related to $\frac{u_{S_k[l]}}{g_{S_k[l]}}$ and we have

$$\varpi_{S_k[l]} > \varpi_{S_n[l]} \Rightarrow \frac{u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}, \mathcal{C}_{[l]}\right)}{g_{S_k[l]}} > \frac{u_{S_n[l]}\left(\boldsymbol{\beta}_{[l]}, \mathcal{C}_{[l]}\right)}{g_{S_n[l]}}$$
$$\Rightarrow \frac{(T-t)\,\epsilon \sum_{P_j \in \mathcal{C}_{[l]}} h_{jk[l]}}{g_{S_k[l]}} > \frac{(T-t)\,\epsilon \sum_{P_j \in \mathcal{C}_{[l]}} h_{nk[l]}}{g_{S_n[l]}}.$$

This means that if the payoffs of two SUs S_k and S_n satisfy $\varpi_{S_k[l]} > \varpi_{S_n[l]}$ in iteration t, this relationship will not change during the following price decreasing process [12]. In other words, each SU has no intention to change its selected sub-band given the sub-bands of the others, which is the definition of the NE of the sub-band allocation game.

Let us now consider results 2) and 3). As mentioned in Section IV, the coalition formation among PUs in different sub-bands can be overlapped. However, if we focus on the coalition formation process within each sub-band l, the grand coalition of PUs can be partitioned into two disjoint coalitions: one is a subset of PUs that can jointly charge the SU in sub-band l and the other is the subset of the rest of PUs. Following the same line as Section IV, let us focus on the possible coalition formed among PUs to decide the price of one SU S_k in a sub-band l. Let us assume $l \in \mathcal{L}_{S_k}$. In this case, the grand coalition $\mathcal{J}$ has been partitioned into two disjoint coalitions: $\mathcal{C}_{[l]}$ and $\mathcal{C}^c_{[l]} = \{P_j : h_{jk[l]} w^*_{S_k[l]} < \underline{q}_{P_j}\}$. Here we abuse the notation and use $\mathcal{C}$ to denote the partition of $\{\mathcal{C}, \mathcal{C}^c\}$ for $\mathcal{C} \cup \mathcal{C}^c = \mathcal{J}$. First, let us prove that the coalition formation in one iteration t of Step 2) in Algorithm 1 is unique, stable and $\triangleright$ maximal for a given pricing vector $\boldsymbol{\beta}_{[l]}$.

Proposition 5: Suppose (2) is satisfied and condition 3) in Proposition 2 holds. For a given $\boldsymbol{\beta}_{[l]} = [\beta_{P_1[l]}, \ldots, \beta_{P_j[l]}]$, the coalition formation achieved by Step 2) in Algorithm 1 is unique, stable and $\triangleright$ maximal.

Proof: From (12), it is observed that, if $\boldsymbol{\beta}_{[l]}(t)$ is fixed, the values of $u_{S_k}(\boldsymbol{\beta}_{[l]}(t), \mathcal{C}_{[l]}(t))$, $w^*_{S_k}(\boldsymbol{\beta}_{[l]}(t))$ and the set of PUs who satisfy (1) are fixed too. Thus, $\mathcal{C}_{[l]}$ is the unique result for the given $\boldsymbol{\beta}_{[l]}(t)$. $\boldsymbol{\beta}_{[l]}(t)$ is also a unique vector for the chosen $\overline{\beta}$ and ϵ. Following the same line as [37], let us now prove that the resulting coalition formation is stable and $\triangleright$ maximal. It can be shown that the resulting coalition $\mathcal{C}_{[l]}(t)$ in iteration t has the following properties,

- P1) For any two disjoint sequential coalitions $\mathcal{C}^1 = \{P_{\tilde{1}}, \ldots, P_{\tilde{j}}\}$ and $\mathcal{C}^2 = \{P_{\widetilde{j+1}}, \ldots, P_{\tilde{l}}\}$ in $\mathcal{C}_{[l]}$ such that $\tilde{j} = |\mathcal{C}^1|, \tilde{l} - \tilde{j} = |\mathcal{C}^2|$ and $\mathcal{C}^1 \cup \mathcal{C}^2 \subseteq \mathcal{C}_{[l]}$, we have $\{\mathcal{C}^1 \cup \mathcal{C}^2\} \triangleright \{\mathcal{C}^1, \mathcal{C}^2\}$;
- P2) For any sequential coalition $\mathcal{C}^3 = \{P_{\tilde{1}}, \ldots, P_{\tilde{j}}\}$ such that $|\mathcal{C}^3| > |\mathcal{C}_{[l]}|$ and $\mathcal{C}^3 \subseteq \mathcal{J}$, we have $\mathcal{C}_{[l]} \triangleright \{\mathcal{C}^3\}$;
- P3) For any non-sequential coalition $\mathcal{C}^4$ such that $\mathcal{C}^4 \subseteq \mathcal{J}$, we have $\mathcal{C}_{[l]} \triangleright \{\mathcal{C}^4\}$.

Property P1 is a direct result of Proposition 2. Property P2 comes from the fact that P_j cannot charge SU S_k if PU $P_j \notin \mathcal{C}_{[l]}$, and hence the contribution of the PU $P_j \in \mathcal{C}^3 \backslash \mathcal{C}_{[l]}$ in the coalition $\mathcal{C}^3$ is always negative (because of the cooperation cost), i.e., $\varpi_{P_j} = -\theta_{P_j} < 0, \quad \forall P_j \notin \mathcal{C}_{[l]}$. To prove Property P3, we observe that if a PU $P_{\tilde{j}}$ for $|C^4| > \tilde{j}$ is not involved in the coalition $\mathcal{C}^4$, following the same line as Property P2 we can claim that the PU $P_{\widetilde{n}}$ for $\widetilde{n} \in \{\tilde{j}, \ldots, |\mathcal{C}^4|\}$ can only provide negative payoff to the coalition and eliminating PU $P_{\widetilde{n}}$ can increase the payoff sum of coalition $\mathcal{C}^4$. Hence, using Properties P1 and P2, we can show that $\mathcal{C}^4$ is always less preferable than $\mathcal{C}_{[l]}$. Properties P1 - P3 include all the possible partitions of PUs for charging the SU in sub-band l. Hence, combining properties P1 - P3 and using the transitive, irreflexive and monotonic properties of $\triangleright$ [37], we can claim that, for all partitions $\mathcal{C}^5 \neq \mathcal{C}_{[l]}$ and $\mathcal{C}^5 \subseteq \mathcal{J}$, $\mathcal{C}_{[l]} \triangleright \mathcal{C}^5$ holds. This concludes the proof. ■

Let us now consider the case that condition 3) in Proposition 2 does not hold. We have the following result.

Proposition 6: If a coalition is formed by a set of PUs that is not sequential, the core of the coalition is empty.

Proof: If condition 3) in Proposition 2 is satisfied, the above result can be directly proved by using Proposition 2. Let us now show that the result still holds if condition 3) in Proposition 2 cannot be satisfied. It is observed in Observation 2 that, for a non-sequential coalition $\mathcal{C}_{[l]}$, we can always find two disjoint subsets $\mathcal{C}^1_{[l]}$ and $\mathcal{C}^2_{[l]}$ such that $\mathcal{C}^1_{[l]} = \{P_{\tilde{1}}, P_{\tilde{2}}, \ldots, P_{\tilde{j}}\}$, $\mathcal{C}^2_{[l]} = \{P_{\widetilde{j+i}}, \ldots, P_{\widetilde{|\mathcal{C}_{[l]}|}}\}$ and $i \geq 2$. In this case, the PUs in $\mathcal{C}^2_{[l]}$ cannot obtain any revenues but increase the cost of cooperation in the coalition $\mathcal{C}_{[l]}$ i.e., we have $R_{\mathcal{C}^2_{[l]}} = 0$, $R_{\mathcal{C}_{[l]}} = R_{\mathcal{C}^1_{[l]}}$ and $\sum_{P_j \in \mathcal{C}_{[l]}} \theta_{P_j}(\mathcal{C}_{[l]}) > \sum_{P_j \in \mathcal{C}^1_{[l]}} \theta_{P_j}(\mathcal{C}^1_{[l]}) + \sum_{P_j \in \mathcal{C}^2_{[l]}} \theta_{P_j}(\mathcal{C}^2_{[l]})$. In other words, we can claim that if $\mathcal{C}_{[l]}$ is not sequential, there always exist at least two subsets $\mathcal{C}^1_{[l]}$ and $\mathcal{C}^2_{[l]}$ such that $v\left(\mathcal{C}^1_{[l]} \cup \mathcal{C}^2_{[l]}\right) < v\left(\mathcal{C}^1_{[l]}, \mathcal{C}^2_{[l]}\right)$. This concludes the proof. ■

From the above proposition, we only need to search for the coalition $\mathcal{C}^*_{[l]}$ using (17) in Algorithm 1 for every sub-band that satisfies sequential conditions. And the main function of (17) and (18) in Algorithm 1 is to search for all the possible subsets of coalition $\mathcal{C}_{[l]}$ that is sequential to find the optimal one.

Let us consider the dynamic coalition updating step in Algorithm 1. It is observed that, in each iteration of Step 3) in Algorithm 1, the PUs form a sequential coalition game. Note that the worth of coalition $\mathcal{C}_{[l]}$ and the transmit power of S_k are functions of $u_{S_k[l]}(\boldsymbol{\beta}_{[l]}, \mathcal{C}_{[l]})$, and dividing the payoff among PUs is unrelated to $v\left(\mathcal{C}_{[l]}\right)$. In addition, as observed in Proposition 1 and (12), both $v\left(\mathcal{C}_{[l]}\right)$ and $\varpi_{S_k[l]}\left(w^*_{S_k[l]}, \boldsymbol{\beta}_{[l]}|\mathcal{C}_{[l]}\right)$ decrease with $u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}, \mathcal{C}_{[l]}\right)$. The main effect of Step 3) in Algorithm 1 is to distributedly decrease the value of $u_{S_k[l]}$ until $w^*_{S_k[l]}$ reaches its upper bound (the interference level increases to reach the interference limit of at least one PU in (2)). From Proposition 1, the resulting $(w^*_{S_k[l]}, \boldsymbol{\beta}^*_{[l]})$ maximizes both the payoff of SU S_k and the payoff sum of $\mathcal{C}_{[l]}$. It is observed that the portions of the revenue charged by each PU to SUs in different sub-bands are independent, and hence maximizing the payoff of the SU S_k and the payoff sum of its corresponding coalition $\mathcal{C}_{[l]}$ in every sub-band is equivalent to maximizing the payoffs of all SUs $S_k \in \mathcal{K}$ and the overall payoff sum of all PUs. Therefore, we can claim that if (13) is satisfied and $\mathcal{C}_{[l]} \neq \emptyset$, $\forall l \in \mathcal{M}$, Algorithm 1 achieves an SE of the hierarchical game with the given $\boldsymbol{\mathcal{L}}_S$. This concludes our proof.

G. Proof of Theorem 2

We can write the Nash bargaining solution in (19) as

$$\max_{\beta_{P_j[l]}} \sum_{P_j \in \mathcal{C}_{[l]}} \log_2\left(\varpi_{P_j[l]}\left(\boldsymbol{w}^*_S, \boldsymbol{\beta}_{[l]}\right) - \varpi^{\min}_{P_j[l]}\right) \quad (31)$$

$$\text{s.t.} \quad \frac{1}{\min\limits_{P_j \in \mathcal{C}_{[l]}}\left\{\frac{\overline{q}_{P_j}}{h_{jk[l]}}\right\} + \frac{1}{g_{S_k[l]}}} \leq u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}\right) < \frac{1}{\max\limits_{P_j \in \mathcal{C}_{[l]}}\left\{\frac{q_{P_j}}{h_{jk[l]}}\right\} + \frac{1}{g_{S_k}}}.$$

The above constraints come from the power constraints in (1) and (2). The objective function in (31) is strictly concave and the constraints are linear, and hence the KKT conditions are necessary and sufficient for the optimal solution.

Hence, we can write the Lagrange multiplier as

$$\begin{aligned} LM = & \sum_{P_j \in \mathcal{C}_{[l]}} \log_2\left(\varpi_{P_j[l]}\left(w^*_S, \beta_{P_j[l]}\right) - \varpi^{\min}_{P_j[l]}\right) \\ & -\lambda_1\left(u_{S_k}\left(\boldsymbol{\beta}_{[l]}\right) - \frac{1}{\min\limits_{P_j \in \mathcal{C}_{[l]}}\left\{\frac{\overline{q}_{P_j}}{h_{jk[l]}}\right\} + \frac{1}{g_{S_k[l]}}}\right) \\ & -\lambda_2\left(\frac{1}{\max\limits_{P_j \in C_{[l]}}\left\{\frac{q_{P_j}}{h_{jk[l]}}\right\} + \frac{1}{g_{S_k[l]}}} - u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}\right)\right) \end{aligned} \quad (32)$$

The first order necessary and sufficient conditions gives

$$\begin{aligned} \frac{\partial LM}{\partial \beta_{P_j[l]}} = & \frac{h_{jk[l]} \min\limits_{P_j \in \mathcal{C}_{[l]}}\left\{\overline{q}_{P_j}/h_{jk[l]}\right\}}{\beta_{P_j[l]} h_{jk[l]} \min\limits_{P_j \in \mathcal{C}_{[l]}}\left\{\overline{q}_{P_j}/h_{jk[l]}\right\} - \hat{\theta}_{P_j[l]}} \\ & - \left(\lambda_1 - \lambda_2\right) h_{jk[l]} = 0 \\ \Rightarrow \ & \beta_{P_j[l]} h_{jk[l]} - \frac{\hat{\theta}_{P_j[l]}}{\min\limits_{P_j \in \mathcal{C}_{[l]}}\left\{\overline{q}_{P_j}/h_{jk[l]}\right\}} = \left(\lambda_1 - \lambda_2\right)^{-1}, \\ & \forall P_j \in \mathcal{C}_{[l]} \end{aligned} \quad (33)$$

where $\hat{\theta}_{P_j[l]} = \theta_{P_j[l]} + \varpi^{\min}_{P_j[l]}$.

Solving (31), we have that the Nash bargaining solution for $\beta_{P_j[l]}$ should satisfy

$$\begin{aligned} & \beta_{P_i[l]} h_{ik[l]} \min_{P_j \in \mathcal{C}_{[l]}}\left\{\overline{q}_{P_j}/h_{jk[l]}\right\} - \hat{\theta}_{P_i[l]} \quad (34) \\ = & \frac{u_{S_k[l]}(\boldsymbol{\beta}_{[l]}) \min\limits_{P_j \in \mathcal{C}_{[l]}}\left\{\overline{q}_{P_j}/h_{jk[l]}\right\} - \hat{\theta}_{\mathcal{C}_{[l]}}}{J}, \ \forall P_j \in \mathcal{C}_{[l]} \end{aligned}$$

where $\hat{\theta}_{\mathcal{C}_{[l]}} = \sum\limits_{P_j \in \mathcal{C}_{[l]}} \hat{\theta}_{P_j[l]}$

Substituting the above result into (31), we have that $\sum\limits_{P_j \in \mathcal{C}_{[l]}} \log_2 \varpi_{P_j[l]}\left(\boldsymbol{w}^*_S, \beta_{P_j[l]}\right)$ is maximized when $u_{S_k[l]}(\boldsymbol{\beta}_{[l]}, \mathcal{C}_{[l]})$ is minimized which is a unique solution given by $u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}\right) = \frac{1}{\min\limits_{P_j \in \mathcal{C}_{[l]}}\left\{\frac{\overline{q}_{P_j}}{h_{jk[l]}}\right\} + \frac{1}{g_{S_k[l]}}}$. Combining (34) and $u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}\right)$, we can derive the result in (20). This completes our proof.

H. Proof of Theorem 3

It can be shown that a sequential coalitional game is always convex, and hence the Shapley value is always in the core. In a sequential coalitional game, the PUs join the coalition $\mathcal{C}_{[l]}$ in an order decided by the power constraints of PUs and the channel gains and hence the possible permutation of the players to join the coalition in 1. Let us write the total payoff of each possible coalition $\mathcal{C}^l \subseteq \mathcal{C}_{[l]}$ as

$$\begin{aligned}
&v\left(\{0\}\right) = 0,\\
&v\left(\left\{P_{\tilde{1}}, P_{\tilde{2}}, \ldots, P_{\widetilde{j-1}}\right\}\right) = 1 - \frac{u_{S_k[l]}\left(\boldsymbol{\beta}_{[l]}\right)}{g_{S_k[l]}}\\
&\quad - \sum_{i=\{\tilde{1},\tilde{2},\ldots,\widetilde{j-1}\}} \theta_{P_{\tilde{i}[l]}}\left(\left\{P_{\tilde{1}}, P_{\tilde{2}}, \ldots, P_{\widetilde{j-1}}\right\}\right)\\
&= \frac{\underline{q}_{P_{\tilde{j}}} g_{S_k[l]}}{\underline{q}_{P_{\tilde{j}}} g_{S_k[l]} + h_{\tilde{j}k[l]}}\\
&\quad - \sum_{i=\{\tilde{1},\tilde{2},\ldots,\widetilde{j-1}\}} \theta_{P_{\tilde{i}[l]}}\left(\left\{P_{\tilde{1}}, P_{\tilde{2}}, \ldots, P_{\widetilde{j-1}}\right\}\right),\\
&\qquad\qquad \forall \widetilde{j-1} < J,\\
&v\left(\mathcal{C}_{[l]}\right) = \frac{g_{S_k[l]} \min\limits_{P_j \in \mathcal{C}_{[l]}} \left\{\frac{\overline{q}_{P_j}}{h_{jk[l]}}\right\}}{g_{S_k[l]} \min\limits_{P_j \in \mathcal{C}_{[l]}} \left\{\frac{\overline{q}_{P_j}}{h_{jk[l]}}\right\} + 1}\\
&\quad - \sum_{i=\{\tilde{1},\tilde{2},\ldots,\tilde{J}\}} \theta_{P_{\tilde{i}[l]}}\left(\left\{P_{\tilde{1}}, P_{\tilde{2}}, \ldots, P_{\tilde{J}}\right\}\right).
\end{aligned}$$

The marginal contribution of total payoff for each PU $P_{\tilde{j}}$ to enter the coalition is given by,

$$\begin{aligned}
\phi_{P_{\tilde{j}}} \;=\; & v\left(\left\{P_{\tilde{1}}, \ldots, P_{\tilde{j}}\right\}\right) - v\left(\left\{P_{\tilde{1}}, \ldots, P_{\widetilde{j-1}}\right\}\right),\\
& \text{for } 1 \leq j \leq J. \qquad (35)
\end{aligned}$$

Hence, $\beta_{P_{\tilde{j}}[l]}$ in (22) is obtained by solving $\frac{\phi_{P_{\tilde{i}}}}{\phi_{P_{\tilde{j}}}} = \frac{h_{\tilde{i}k[l]}\beta_{P_{\tilde{i}}[l]}}{h_{\tilde{j}k[l]}\beta_{P_{\tilde{j}}[l]}}$ for $v\left(\mathcal{C}_{[l]}\right) \geq 0$. This concludes our proof.

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