Bayesian Hierarchical Mechanism Design for Cognitive Radio Networks

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***Abstract*—This paper considers a cognitive radio network where the licensed network, referred to as the primary user (PU) network, consists of a hierarchical structure in which multiple operators coexist in the same coverage area where each of the operators controls an exclusive set of frequency sub-bands. Unlicensed users, referred to as the secondary users (SU), first send their requests to the operators, and can only access the sub-bands controlled by the operators that accept their requests. SUs are selfish and cannot exchange private information with each other. We model the dynamic spectrum access (DSA) problem of the SUs as a Bayesian game, referred to as the DSA game. We model the PU network as a forest where the roots represent the operators and the leaves represent the operators' sub-bands. We propose a novel forest matching market to model the interaction between the SUs and the PU network. In this market, a set of SUs can be first matched to a set of operators and the SUs matched to the same operator can then be matched to the corresponding sub-bands. We propose a distributed algorithm that results in a stable forest matching structure which coincides with the optimal Bayesian Nash equilibrium of the DSA game. We prove that the Bayesian hierarchical mechanism associated with our proposed algorithm incentivizes truth-telling by SUs. Our algorithm does not require each SU to know the preference and conflict-solving rule of the PU network or the payoffs and actions of other SUs, and the complexity of each iteration in the worst case is given by $O(L^2N^2K)$ where L is the number of operators, N is the maximum number of sub-bands of each operator and K is the number of SUs.**

***Index Terms*—Cognitive radio, stable matching, forest matching, stable marriage, college admission, dynamic spectrum access, hierarchical, game theory, mechanism design, Bayesian game.**

I. Introduction

Many of today's networking systems operate according to a hierarchical structure. For example, in telecommunication networks, smart grids, cloud storage systems, etc., multiple operators (e.g., telecommunication operators, electricity companies, cloud storage providers, etc.) coexist in the same service area. Each operator has been licensed exclusive use of a resource (e.g., frequency bands, electrical grid, or cloud storage infrastructure) which can be further divided into resource blocks. Each resource block can then be used to provide an individual service, such as voice/video call, electricity supply, data storage, etc., for a user (e.g., mobile service subscriber, electrical appliance, storage application, etc.), referred to as the primary user (PU). If a set of unlicensed user equipments, referred to as secondary users (SUs), can intelligently access this resource without causing intolerable performance degradation to the PUs, the system will turn into a licensed resource sharing (LRS) networking system. In this system, SUs cannot access any resource block unless they obtain permission from the corresponding operator.

In this paper, we focus on a cognitive radio (CR) network in which the spectrum licensed to network operators and their licensed subscribers (PUs) can be dynamically accessed by the unlicensed subscribers (SUs) according to different service requirements and environments [1]. We refer to the licensed networking system consisting of operators and their corresponding sub-bands and subscribers as *the PU network*. We also use the terms *frequency band* (or *spectrum*) and *sub-band* to denote the resource licensed to each operator and the resource block that can be allocated to each subscriber, respectively.

We study the dynamic access of a set of SUs to the spectrum licensed to a PU network with a hierarchical structure consisting of multiple operators and their corresponding sub-bands. The hierarchical structure of the PU network makes it difficult for SUs to decide which operators' sub-bands they want to access. More specifically, each SU needs to send a request to an operator before accessing any sub-band. Once its request has been accepted, the SU will stick to the operator that accepts its request for a certain period of time. Each operator only holds a license for a limited number of sub-bands, and hence can only allow a limited number of SUs to access its spectrum. If the number of SUs requesting the resources of the same operator exceeds the limit for this operator, a conflict happens. The operator and sub-band finally allocated to each SU depend not only on the preference of the SU over the operators and sub-bands, but also on rules applied by the PU network to solve conflicts and on the benefits each operator can

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obtain from the population of SUs. For example, suppose one SU believes a sub-band controlled by an operator can provide the maximum performance and sends its request to this operator. However, the request of the SU can be rejected by the operator. Or even if the SU's request is accepted, its request for the performance-maximizing sub-band may be rejected because the operator allocates this sub-band to another competing SU. In this case, another sub-band of the operator will be allocated to the SU, which may result in performance that is even worse than that achieved by accessing sub-bands of other operators. In other words, selecting the operator with the performance-maximizing sub-band may not always the best choice for the SUs. Furthermore, it has been observed in [2]–[5] that there is no stable mechanism that can prevent all SUs with a general domain of preferences from obtaining higher benefits by misrepresenting their true private information. Therefore, how to design a truth-telling mechanism that can also result in the optimal social choice for the SUs is a challenging and important problem.

It can be observed that the interaction among SUs and that between SUs and PUs play a dominant role in determining the performance of SUs and the PU network [6]–[8]. This motivates us to apply game theory to analyze the dynamic spectrum access problem. However, one of the most important solution concepts in game theory, the *Nash equilibrium*, is generally not unique or optimal [9]. For example, it has been shown in [10] that if we model the spectrum access problem as a one-stage non-cooperative game, any sub-band allocation scheme is a Nash equilibrium. Furthermore, achieving the optimal Nash equilibrium is not always possible, or if possible, it may be an NP-hard problem [9], [11].

We consider the distributed optimization of the dynamic spectrum access (DSA) problem for SUs in a CR network. In this problem, SUs can access the spectrum licensed to multiple operators that coexist in the area of interest. To maximize their payoffs, the SUs will first decide their preferred operators, and then compete for the limited number of sub-bands of each operator. Each SU cannot know the preference and the conflict-solving rule of operators, or observe the private information such as preferences and payoffs of other SUs. This motivates us to model the DSA problem as a Bayesian non-cooperative game, referred to as the *DSA game*. We seek a self-enforcing truth-telling mechanism design that can incentivize all SUs to decide their actions based on their true preference and can eventually result in a unique and optimal Bayesian Nash equilibrium. It is observed that the mechanism design not only includes developing rules for the operators to solve the conflicts when the number of requesting SUs exceeds the limit but also requires establishing policies for strategic SUs to distributedly compete for the limited number of operators and sub-bands. To solve the above problem, we model the PU network as a forest structure [12] and propose a novel forest matching market to model the interaction between the SUs and the PU network. The DSA problem of SUs can then be modeled as a matching problem between a set of SUs and a forest consisting of a set of roots (operators) and their corresponding leaves (sub-bands). We develop a distributed Bayesian hierarchical algorithm that, despite the private information and selfish behavior of SUs, can result in a unique and stable forest matching structure which coincides with the optimal Bayesian Nash equilibrium of the DSA game. Our proposed solution contains two separate algorithms for SUs to choose their operators and sub-bands and a Bayesian belief updating algorithm. We also prove that the associated Bayesian hierarchical mechanism incentivizes all SUs to select the operators and sub-bands based on their true private information.

Let us briefly summarize the main contributions of the paper as follows:

1) We formulate a Bayesian non-cooperative game-based framework to model the DSA problem of SUs in a CR network.
2) We introduce a novel Bayesian hierarchical mechanism design framework to approach the unique and optimal Bayesian Nash equilibrium of our DSA game.
3) We propose a novel stable forest matching algorithm to achieve a stable matching between a set of SUs and a hierarchical PU network which coincides with the optimal Bayesian Nash equilibrium of our DSA game.
4) We present numerical results to assess the performance of the proposed methods under different situations.

The rest of this paper is organized as follows. The background and related work are reviewed in Section II. The network model is introduced in Section III. We establish the DSA game in Section IV. We introduce the forest matching market and the Bayesian hierarchical mechanism in Section V. Extensions and future works are discussed in Section VI. The numerical results are presented in Section VII, and the paper is concluded in Section VIII.

II. Background and Related Work

Game theory has been widely applied to analyze the performance of CR networks. More specifically, in [6], [8], [13], [14], the sub-band allocation problem for CR networks has been modeled as a non-cooperative game to study the interaction among the competing SUs. Coalitional game theoretic models have been applied to study the interaction among the cooperative users in CR networks in [15], [16]. A detailed survey of applications of game theory to CR networks has been presented in [17]–[20].

Although game theory has been shown to be an effective tool to study and analyze the interaction among individual players, it is known that its outcomes such as the Nash equilibrium solution, are generally not unique or optimal. This motivates *mechanism design*, whose main

objective is to lead the institutions governing the interactions of a game to implement a socially desirable solution [21]. In [22], [23], an auction mechanism has been applied to CR networks where cheating between wireless devices has been avoided by allowing payments received by each wireless device to be freely transferred. However, in many practical wireless systems, payment/money transferring between SUs is unrealistic [24]–[26]. In this paper, we focus on designing a *mechanism without monetary exchange* where each SU only cares about its own private payoff and there is no information exchange among SUs. To the best of our knowledge, this is the first work to consider mechanism design without monetary exchange for CR networks.

We propose a novel forest matching market which can be regarded as a generalization of the traditional two-sided matching market (also called stable marriage market [27], bipartite matching market [28], [29]). The two-sided stable matching problem has been widely studied from both theoretical and practical perspectives [27], [28], [30]–[32]. In this problem, each agent belonging to one side of the market has a preference over the agents of the other side, and tries to find a matching that can optimize its performance. Many extensions of these problems have been studied in the literature. More specifically, the case of some agents on one side with preferences over a sub-set of the agents on the other side has been studied in [33]. The case that the agents from one side can have equal preference over multiple agents of the other side, also called stable marriage with tie, has been studied in [34]. Empirical studies of the different variations of the two-sided matching problem have been reported in [31], [35]. A survey of the stable marriage market and its variants has been presented in [36].

Different from the existing works, we consider the case in which a set of agents from one side consists of a forest structure. We focus on the matching problem between the set of agents of one side and the roots and leaves in a forest of the other side. To the best of our knowledge, this is the first work to study the matching problem between a set of agents and a forest.

III. Network Model

We consider a CR network in which a set of K SUs $\mathcal{D} = \{D_1, D_2, \ldots, D_K\}$ share the spectrum held by a set of L co-located network operators $\mathcal{O} = \{1, 2, \ldots, L\}$ as shown in Figure 1. Each operator i has been licensed an exclusive set of sub-bands, labeled as $\mathcal{S}_i = \{S_i^1, S_i^2, \ldots, S_i^{N_i}\}$ where N_i is the number of sub-bands licensed to operator i and $\mathcal{S}_i \cap \mathcal{S}_j = \emptyset$ for $i \neq j$ and $i, j \in \mathcal{O}$. We also denote $\mathcal{S} = \bigcup_{i=1}^{L} \mathcal{S}_i$. We label the PU currently occupying sub-band S_i^l as P_i^l for $P_i^l \neq 0$. If there is no PU using sub-band S_i^l, we have $P_i^l = 0$. Let $\mathcal{P}_i$ be the set of all PUs in operator i, i.e., $\mathcal{P}_i = \{P_i^l : P_i^l \neq 0, \forall\, l \in \{1, 2, \ldots, N_i\}\}$. We list the main notation adopted in this paper in Table I.

We assume that each sub-band (either occupied or unoccupied by a PU) can be accessed by at most one SU.

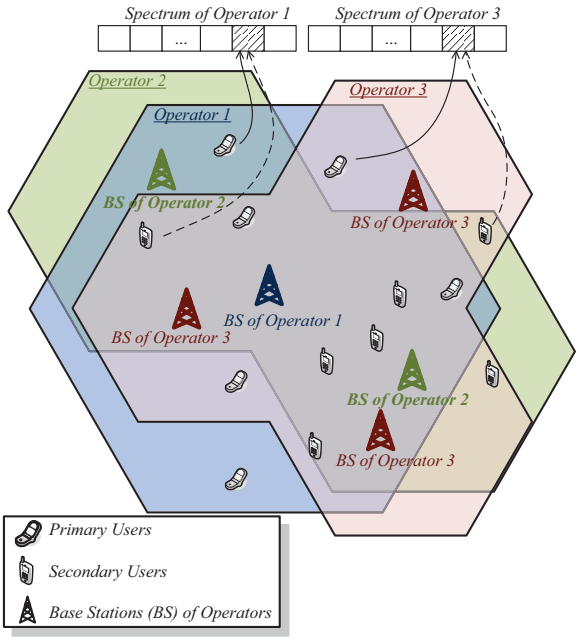

Fig. 1. A CR network with 3 operators, 7 SUs and 6 PUs.

This assumption is reasonable because imposing the limit of one SU to share each sub-band allows the operator to evaluate and control the interference caused by SUs. For example, if either the PU or SU, or both, observes higher-than-tolerable interference, the operator can simply remove the SU from the sub-band. If two or more SUs share the same sub-band, it will be difficult to evaluate which SU causes the highest interference to the PU network, or which SU has to be removed, and it is generally inefficient to simultaneously remove all SUs from the sub-band whenever an operator observes high interference. Our model can be extended to the case with more than one SU sharing each sub-band. We will provide a more detailed discussion in Section VI.

We consider the following power constraints in each sub-band S_i^l of operator i,

$$h_{P_i^l D_k} w_{D_k} \quad \leq \quad \bar{Q}_i^l, \text{ if } P_i^l \neq 0, \tag{1}$$

where $h_{P_i^l D_k}$ is the channel gain between PU P_i^l and SU D_k in sub-band S_i^l, w_{D_k} is the transmit power of D_k and $\bar{Q}_i^l$ is the maximum tolerable interference level for sub-band S_i^l. If an SU D_k cannot satisfy the above constraint, it will be excluded from sub-band S_i^l.

Let the payoff obtained by each SU D_k in sub-band S_i^l be $\varpi_{D_k}[S_i^l]$ for $S_i^l \in \mathcal{S}_i$. We consider a general model and the payoff of each SU can be any performance measure or function generated from its received signal to interference plus noise ratio (SINR). For example, if the SU wants to maximize its transmit rate per bandwidth price, the payoff function of SU D_k when it accesses sub-band S_i^l can be written as

$$\varpi_{D_k}[S_i^l] = \frac{\rho_i^l}{e\left(\rho_i^l\right)} \log\left(1 + SINR_{D_k}[S_i^l]\right), \tag{2}$$

TABLE I
LIST OF NOTATIONS

Symbol	Definition
L	Number of network operators
K	Number of SUs
$\mathcal{D}$	Set of SUs
$\mathcal{O}$	Set of operators
$\mathcal{S}_i$	Set of available sub-bands for operator i
$\mathcal{P}_i$	Set of PUs in operator i
$\mathcal{R}^o_{D_k}$	Preference of SU D_k over the operators
$\mathcal{R}^b_{D_k}$	Preference of SU D_k over the sub-bands of its chosen operator
$\mathcal{R}_{D_k}$	Preference of SU D_k
$\mathcal{R}_i$	Preference of operator i over the SUs
$\tilde{i}_{D_k}$	The ith most preferred operator for SU D_k
$\tilde{S}^n_{D_k}$	The nth most preferred sub-band for SU D_k controlled by its chosen operator
N_i	Number of sub-bands of operator i
D_k	The kth SU
S^l_i	The lth sub-band of operator i
P^l_i	The PU of operator i occupying sub-band S^l_i
$\mathcal{U}_i$	Set of SUs accepted by operator i
q_i	Maximum number of SUs that can be accepted by operator i
Γ^o	Conflict-solving rule of the operators
Γ^b	Conflict-solving rule of each sub-band of the operators
b_{D_k}	Belief function of SU D_k about actions of other SUs
$\mathcal{H}$	Forest structure of the PU network
$\varpi_{D_k}[S^l_i]$	Payoff of SU D_k when it accesses sub-band S^l_i of operator i
$\bar{\varpi}_{D_k}\left(a_{D_k}, b_{D_k}\right)$	Average payoff of SU D_k with action a_{D_k} and belief function b_{D_k}
$c\left(\boldsymbol{y}\right)$	Social choice function with the given type $\boldsymbol{y}$ of SUs

where ρ^l_i is the bandwidth of sub-band S^l_i and $e\left(\rho^l_i\right)$ is the price of bandwidth paid to the operator i and $SINR_{D_k}\left[S^l_i\right]$ is the signal to noise ratio received at SU D_k in sub-band S^l_i, given by

$$SINR_{D_k}[S^l_i] = \begin{cases} \frac{h_{D_k[S^l_i]} w_{D_k}}{\sigma_{D_k[S^l_i]} + h_{P^l_i D_k} w_{P^l_i}}, & \text{if } P^l_i \neq 0, \\ \frac{h_{D_k[S^l_i]} w_{D_k}}{\sigma_{D_k[S^l_i]}}, & \text{if } P^l_i = 0, \end{cases} \tag{3}$$

where $h_{D_k[S^l_i]}$ is the channel gain between the source and destination of SU D_k in sub-band S^l_i, $\sigma_{D_k[S^l_i]}$ is the additive noise received by SU D_k in sub-band S^l_i, and $w_{P^l_i}$ is the transmit power of P^l_i. We have $\varpi_{D_k}[S^l_i] \neq \varpi_{D_k}[S^n_m]$ for $S^l_i \neq S^n_m$ and $S^l_i, S^n_m \in \mathcal{S}$.

The revenue $\eta_{S^l_i}[D_k]$ obtained by operator i by allowing SU D_k to access sub-band S^l_i can be a function of the resulting interference. For example, if the revenue $\eta_{S^l_i}[D_k]$ is a linear function of the received interference at PU P^l_i when $P^l_i \neq 0$ or the SINR of D_k when $P^l_i = 0$, we have[1]

$$\eta_{S^l_i}[D_k] = \begin{cases} \beta^i h_{D_k P^l_i} w_{D_k}, & \text{If } P^l_i \neq 0, \\ \beta^i \frac{h_{D_k[S^l_i]} w_{D_k}}{\sigma_{D_k[S^l_i]}}, & \text{If } P^l_i = 0, \end{cases} \tag{4}$$

where β^i is the pricing coefficient of operator i. If it is clear from the context that D_k obtains a sub-band from operator i, we use $\eta_i[D_k]$ to denote the revenue of operator i obtained from SU D_k.

As mentioned previously, each operator i has a limited number of sub-bands, and hence can only provide services to a limited number of SUs. We refer the maximum number of SUs an operator i can accept as its *quota*, denoted as q_i. Note that $q_i \leq N_i$ needs to be satisfied when each sub-band can be occupied by at most one SU. However, if we allow multiple SUs to share each sub-band, we will have $q_i > N_i$. We will discuss this case in detail at Section VI. In this paper, we set $q_i = N_i$ for $i \in \mathcal{O}$. When the number of SUs requesting permission to access the spectrum of operator i exceeds q_i, a *conflict* will happen. In this case, only q_i SUs will be accepted and the remaining SUs will be rejected and excluded from the spectrum of operator i. These rejected SUs will then send their requests to other operators. The process will continue until all SUs have been allocated operators. Similarly, if a set of SUs, labeled as $\mathcal{U}_i$, has been accepted by operator i, these SUs will then compete for the set $\mathcal{S}_i$ of sub-bands of operator i. If at least two SUs in $\mathcal{U}_i$ choose the same sub-band, only one of them will be allowed to access this sub-band. The rest of the requesting SUs will have to compete for the remainder of sub-bands in $\mathcal{S}_i$. We assume SUs are selfish and always try to maximize their payoff. Each SU can establish and maintain a *preference*, a ranked list, over the operators and their corresponding sub-bands. Let the preference of each SU D_k over the operators and sub-bands be $\mathcal{R}^o_{D_k}$ and $\mathcal{R}^b_{D_k}$, respectively. Note that $\mathcal{R}^o_{D_k}$ and $\mathcal{R}^b_{D_k}$ are closely related to each other. For example, consider a CR network with two operators 1 and 2. If an SU D_k believes that the sub-band it can obtain from operator 2 can provide a higher payoff than that from operator 1, the preference of SU D_k over operators is given by $\mathcal{R}^o_{D_k} = \langle 2, 1\rangle$. If we use $\tilde{i}_{D_k}$ to denote the ith most preferred operator for SU D_k, we can rank the operators from the most to the least preferred ones for SU D_k and write its preference as $\mathcal{R}^o_{D_k} = \langle \tilde{1}_{D_k}, \tilde{2}_{D_k}\rangle$ where $\tilde{1}_{D_k} = 2$ and $\tilde{2}_{D_k} = 1$ in this example. Similarly, if we use $\tilde{S}^n_{D_k}$ to denote the nth most preferred sub-band for SU D_k of its chosen operator i, we can write the preference of $D_k \in \mathcal{U}_i$ over the set $\mathcal{S}_i$ of sub-bands as $\mathcal{R}^b_{D_k} = \langle \tilde{S}^1_{D_k}, \tilde{S}^2_{D_k}, \ldots, \tilde{S}^{N_i}_{D_k}\rangle$. We can also write the

[1]Applying a linear function of the resulting interference as the pricing function of the operators has been adopted in [6], [8], [37], [38]. Applying a linear function of SINR for each SU as the pricing function is motivated by the fact that many existing telecommunication mobile systems charge according to their communication data rates, which are monotonically increasing functions of SINR. Note that, in these pricing functions, the operator can control the interference of SUs to the PU network by adjusting the value of pricing coefficient β^i [6].

preference of each SU D_k as $\mathcal{R}_{D_k} = \langle \mathcal{R}^o_{D_k}, \mathcal{R}^b_{D_k} \rangle$. We denote $\boldsymbol{\mathcal{R}}^o = \{\mathcal{R}^o_{D_k}\}_{D_k \in \mathcal{D}}$, $\boldsymbol{\mathcal{R}}^b = \{\mathcal{R}^b_{D_k}\}_{D_k \in \mathcal{D}}$ and $\boldsymbol{\mathcal{R}} = \{\mathcal{R}_{D_k}\}_{D_k \in \mathcal{D}}$. We assume that each operator i will only release sub-band information to the SUs that have been given permission to access its spectrum. In other words, an SU D_k can only establish the preference $\mathcal{R}^b_{D_k}$ over the sub-bands of operator i if the request sent by SU D_k has been accepted by operator i.

The operator and sub-band finally allocated to each SU D_k not only depend on the preference of SU D_k but also relate to the conflict-solving rules employed by the operators and the payoffs and preferences of other SUs. Therefore, establishing a proper rule for the operators to accept or reject the requests of SUs is very important. We consider distributed optimization for the DSA problem in a CR network and assume SUs cannot exchange information with each other or know the preference or conflict-solving rules of PUs.

Since each SU will send its request signal to the operators before accessing any sub-band, each operator can use the received request signal to evaluate the resulting revenues and establish its preference over the requesting SUs. In our model, different operators can belong to different network systems and hence cannot communicate or exchange information such as the preference and the revenues with each other. Note that each operator needs to first decide whether to grant permission to access its spectrum to SUs before knowing which specific sub-bands will be requested by each SU. In this paper, we mainly focus on the dynamic spectrum access (DSA) of SUs, and assume the preference $\mathcal{R}_i$ of each operator i over the SUs has been established based on a predefined criterion unrelated to the final sub-band allocated to each SU[2]. For example, if the operators try to maintain a guaranteed worst case revenue from SUs, the preference of each operator j will be a list of SUs in the order of the guaranteed minimum revenue obtained when each SU accesses the sub-band providing the minimum revenue, i.e., the preference of operator i over SUs is given by $\mathcal{R}_i = \{\tilde{D}_i^1, \tilde{D}_i^2, \ldots, \tilde{D}_i^K\}$ where $\tilde{D}_i^k$ is the kth most preferred SU of operator i and, for any two SUs $\tilde{D}_i^n, \tilde{D}_i^k \in \mathcal{D}$, $n < k$ if $\eta_i^{\min}[D_n] > \eta_i^{\min}[D_k]$, where $\eta_i^{\min}[D_k] = \min\limits_{S_i^l \in \mathcal{S}_i} \left\{\eta_{S_i^l}[D_k]\right\}$, $\forall D_k \in \mathcal{D}$.

After a set of SUs obtains permission to access sub-bands of operator i, they will compete with each other for the set of sub-bands $\mathcal{S}_i$. In this case, operator i can also establish a preference $\mathcal{R}_{S_i^l}$ for each of its sub-band S_i^l over all the accepted SUs[3], i.e., the preference for sub-band S_i^l, denoted by $\mathcal{R}_{S_i^l} = \{\tilde{D}^1_{S_i^l}, \tilde{D}^2_{S_i^l}, \ldots, \tilde{D}^{|\mathcal{U}_i|}_{S_i^l}\}$, should satisfy that, for any two SUs $\tilde{D}^n_{S_i^l}, \tilde{D}^k_{S_i^l} \in \mathcal{U}_i$, $n < k$ means $\eta_{S_i^l}[D_n] > \eta_{S_i^l}[D_k]$.

In a CR network, each SU needs to request permission from an operator before accessing its sub-bands. In many practical systems, once the request has been accepted, an agreement (e.g., a contract) between the SUs and the operators that accept their requests will be enforced to avoid SUs changing their decisions within a short period of time and also to prevent the operators from retracting the allocated sub-bands from SUs within the agreed time period. For example, in a telecommunication system, a mobile service subscriber needs to sign a fixed-term contract with a telecommunication operator and cannot switch to another operator within the contract period. In this paper, we assume the communication time can be divided into time slots. At the beginning of each time slot, all SUs decide which operators and sub-bands to send their request to. The SUs cannot change their decisions during the rest of each time slot, but can switch to different operators and sub-bands in different time slots. In the rest of this paper, we focus on designing a mechanism that can incentivize SUs to make decisions based on their true preference and eventually approach a socially desirable outcome. It is generally unrealistic to assume each SU can predict the type information of other SUs instantaneously before it makes decisions at the beginning of each time slot. It is however possible for each SU to eavesdrop on the past decisions of other SUs. Therefore, we assume each SU can observe the decisions of other SUs in the previous time slots. Note that the belief and the decision of each SU in the current time slot are private information that is nnot be known by other SUs.

IV. A Bayesian Game Framework and Mechanism Design for DSA Problem

A. A Bayesian Game Framework

We model the DSA problem of SUs as a Bayesian game, referred to as the *DSA game*, which is formally defined as follows:

Definition 1: The *DSA game* is defined by a tuple $\mathcal{G} = \langle \mathcal{D}, \boldsymbol{\mathcal{A}}, \boldsymbol{\mathcal{T}}, \boldsymbol{B}, \bar{\boldsymbol{\varpi}} \rangle$ where $\mathcal{D}$ is the set of players (i.e., SUs), $\boldsymbol{\mathcal{A}} = \{\mathcal{A}_{D_k}\}_{D_k \in \mathcal{D}}$ is the action space of SUs, $\boldsymbol{\mathcal{T}} = \{\mathcal{T}_{D_k}\}_{D_k \in \mathcal{D}}$ is the type space of SUs, $\boldsymbol{B} = \{B_{D_k}\}_{D_k \in \mathcal{D}}$ is the belief function of SUs about types of others, and $\bar{\boldsymbol{\varpi}}$ is the payoff of SUs.

In DSA game, action $a_{D_k} = \langle a^o_{D_k}, a^b_{D_k} \rangle$ of each SU D_k can be divided into two parts: operator selection

[2]This setting has already been applied in many practical systems. For example, in college admission systems, many universities have general admission requirements, such as SAT score and high school transcripts, for accepting students [39]. These requirements are unrelated to which departments or programmes the students finally choose.

[3]As observed in Section III, to maintain the QoS of the existing PUs, the sub-band sharing between SUs and PUs needs to be strictly controlled by the operators. Therefore, the preference of each sub-band over the SUs has to be established and maintained by the operators. To simplify our discussion, in this paper, we use the term "preference of sub-band" to denote the preference of the operator over the access by the SUs to each of its sub-bands.

action $a^o_{D_k} \in \mathcal{O} \cup \{\emptyset\}$ specifies which operator SU D_k will send its request to, and sub-band selection action $a^b_{D_k} \in \mathcal{S} \cup \{\emptyset\}$ specifies which sub-band SU D_k will request after being granted permission to access the spectrum of an operator. We can write set $\mathcal{A}_{D_k}$ of the possible actions of each SU D_k as $\mathcal{A}_{D_k} = \mathcal{O} \cup \{\emptyset\} \times \mathcal{S} \cup \{\emptyset\}$. For example, $a^o_{D_k} = i$ means SU D_k decides to send request to operator i. If $a^o_{D_k} = \emptyset$, it means D_k does not send a request to any operator. We also use $a^b_{D_k} = S^l_i$ to mean that D_k will send a request for sub-band S^l_i after being accepted by operator i. Similarly, we use $a^b_{D_k} = \emptyset$ to mean D_k does not send a request for any sub-band. We denote $\boldsymbol{a}^o = \{a^o_{D_k}\}_{D_k \in \mathcal{D}}$, $\boldsymbol{a}^b = \{a^b_{D_k}\}_{D_k \in \mathcal{D}}$ and $\boldsymbol{a} = \{a_{D_k}\}_{D_k \in \mathcal{D}}$. The request sent by each SU to the operators for sub-bands can be rejected according to the operator's conflict-solving rules. We define the *conflict-solving rule of operators* as a function Γ^o such that $\Gamma^o(D_k, \boldsymbol{a}^o) = i$ means that operator i accepts the request sent by SU D_k. If D_k cannot obtain permission to access the spectrum of any operator, we write $\Gamma^o(D_k, \boldsymbol{a}^o) = D_k$. Similarly, we define the *conflict-solving rule of sub-bands*[4] as a function Γ^b such that $\Gamma^b(D_k, \boldsymbol{a}^b) = S^l_i$ means that D_k has been given permission to access sub-band S^l_i of operator i. We use $\Gamma^b(D_k, \boldsymbol{a}^b) = D_k$ to mean that D_k cannot access any sub-band of its operator. We denote the final operator and sub-band allocation structure as $\Gamma = \langle \Gamma^o, \Gamma^b \rangle$, i.e., $\Gamma(D_k, \boldsymbol{a}) = \langle i, S^l_i \rangle$ means D_k has been assigned to operator i and sub-band S^l_i. The type $y_{D_k} \in \mathcal{T}_{D_k}$ of each SU D_k which includes its preference over the possible actions is private information and can only be known by itself. Each SU cannot know the types of other SUs but can establish a belief function about actions of other SUs using its previous observations. Since each SU decides its action using its type, we can define a strategy function f_{D_k} for each SU D_k to map its type into an action, i.e., $f_{D_k} : \mathcal{T}_{D_k} \to \mathcal{A}_{D_k}$. In this way, we can convert the belief $B_{D_k}\left(\boldsymbol{y}_{-D_k}\right)$ of each SU D_k about the types of their SUs into the belief about the actions of other SUs, denoted as $b_{D_k}\left(\boldsymbol{a}_{-D_k}\right)$. From the previous discussion, it can be observed that the final operator and sub-band allocation structure can be determined by the action profile of all SUs and the conflict-solving rules of operators and sub-bands. We can hence write the expected payoff of each SU D_k achieved by its action a_{D_k} and belief $b_{D_k}\left(\boldsymbol{a}_{-D_k}\right)$ for the given conflict-solving rules of operators and sub-bands as

$$\begin{aligned} &\bar{\varpi}_{D_k}\left(a_{D_k}, b_{D_k}\left(\boldsymbol{a}_{-D_k}\right)\right) \\ &= \sum_{\boldsymbol{a}_{-D_k} \in \boldsymbol{\mathcal{A}}_{-D_k}} b_{D_k}\left(\boldsymbol{a}_{-D_k}\right) \varpi_{D_k}\left[\Gamma^b\left(D_k, \boldsymbol{a}^b\right)\right], \end{aligned} \tag{5}$$

where $-D_k$ denotes all SUs except D_k and $\varpi_{D_k}\left[\Gamma^b\left(D_k, \boldsymbol{a}^b\right)\right]$ is the payoff obtained by D_k when accessing sub-band $\Gamma^b\left(D_k, \boldsymbol{a}^b\right)$ defined in (2). Note that $\Gamma^b\left(D_k, \boldsymbol{a}^b\right)$ is the result of the action profile of all SUs as well as the conflict-solving rules of both operators and sub-bands.

[4]In most existing wireless networking systems, the operator controls the sub-band usage. Therefore, in this paper, we use the term "conflict-solving rule of sub-band" to mean the acceptance and rejection decision of the operator about the request sent by SUs for each of its sub-bands.

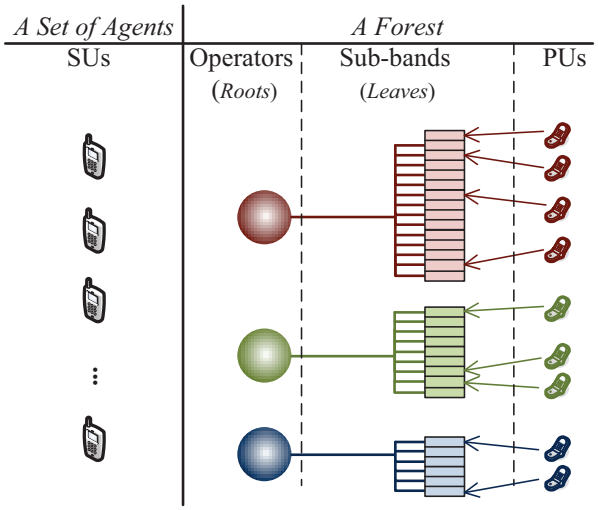

Fig. 2. A forest matching structure for the CR network given in Figure 1.

We consider a (finitely) repeated game setting in which each SU can learn from the resulting payoffs and the observations of the previous time slots and update its belief function at the end of each time slot. Each SU will then use the updated belief function to decide its preference and action for the next time slot.

The main solution concept in our proposed DSA game is the Bayesian Nash equilibrium, which is formally defined as follows:

Definition 2: A *Bayesian Nash equilibrium* of the DSA game is an action profile $\boldsymbol{a}^* = \langle a^*_{D_k} \rangle_{D_k \in \mathcal{D}}$ such that

$$\begin{aligned} &\bar{\varpi}_{D_k}\left(a^*_{D_k}, b_{D_k}\left(\boldsymbol{a}^*_{-D_k}\right)\right) \geq \bar{\varpi}_{D_k}\left(a_{D_k}, b_{D_k}\left(\boldsymbol{a}^*_{-D_k}\right)\right), \\ &\forall a_{D_k} \in \mathcal{A}_{D_k} \text{ and } D_k \in \mathcal{D}. \end{aligned} \tag{6}$$

A Bayesian Nash equilibrium $\boldsymbol{a}^*$ is said to be *(Pareto) optimal* if there is no other Bayesian Nash equilibrium $\boldsymbol{a}'$ such that $\bar{\varpi}_{D_k}\left(a^*_{D_k}, b_{D_k}\left(\boldsymbol{a}^*_{-D_k}\right)\right) \leq \bar{\varpi}_{D_k}\left(a'_{D_k}, b_{D_k}\left(\boldsymbol{a}'_{-D_k}\right)\right)\ \forall D_k \in \mathcal{D}$ where the inequality holds strictly for at least one SU.

B. Bayesian Mechanism Design

It has been observed [11], [30] that for an unrestricted domain of the action profiles of SUs, at least one SU can always improve its performance by misrepresenting its true type, assuming other SUs tell the truth. We hence also seek a mechanism that prevents SUs from obtaining benefits by "cheating" on their strategies. We define a set $\boldsymbol{\Lambda}$ of alternatives (candidates) as the set of all possible operator and sub-band allocations of SUs. As mentioned previously, for a given conflict-solving rule, each action profile of SUs will result in a sub-band allocation scheme for SUs, i.e., we use $\lambda \in \boldsymbol{\Lambda}$ to denote an operator and

sub-band allocation scheme. Let us define the social choice function as follows:

Definition 3: A *social choice function* $c : \boldsymbol{\mathcal{T}} \rightarrow \boldsymbol{\Lambda}$ is a mapping from the type of all SUs to a single candidate of the social choice.

The social choice function specifies the possible resulting outcome of our DSA game with each type profile $\boldsymbol{y}$ of SUs. Let us define the Bayesian mechanism as follows:

Definition 4: [11, Chapter 9.3.2] [21, Chapter 4.1] A *Bayesian (direct revelation) mechanism* for the DSA game is given by the type space $\boldsymbol{\mathcal{T}}$, belief function $\boldsymbol{b}$, action space $\boldsymbol{\mathcal{A}}$ defined in Definition 1, an alternative set $\boldsymbol{\Lambda}$, payoff $\bar{\varpi}_{D_k}$ for each SU D_k and an outcome function $u : \boldsymbol{\mathcal{A}} \rightarrow \boldsymbol{\Lambda}$. A Bayesian mechanism implements a social choice function c if for some Bayesian Nash equilibrium $\boldsymbol{a}^*$ of DSA game, we have $c(\boldsymbol{y}) = u(\boldsymbol{a}^*)$ for all $\boldsymbol{y} \in \boldsymbol{\mathcal{T}}$. A Bayesian mechanism is said to be *incentive compatible* (also called *strategy-proof* or *truthful*) if the social choice function $c(\boldsymbol{y}) = \lambda$ satisfies the following condition for all SUs,

$$\bar{\varpi}_{D_k}\left(a^*_{D_k}, b_{D_k}\left(\boldsymbol{a}_{-D_k}\right)\right) \geq \bar{\varpi}_{D_k}\left(a_{D_K}, b_{D_k}\left(\boldsymbol{a}_{-D_k}\right)\right), \quad \forall D_k \in \mathcal{D}, a^*_{D_k}, a_{D_k} \in \mathcal{A}_{D_k}, \text{and } \boldsymbol{a}_{-D_k} \in \boldsymbol{\mathcal{A}}_{-D_k}.$$

In this paper, we seek a mechanism that plays the role of an *invisible hand*, that is, when SUs interact through the mechanism, despite the private information, self-interested and autonomous behavior of the SUs, they will have an incentive to make their decisions based on their true type information, which eventually leads to the optimal Bayesian Nash equilibrium of the DSA game.

V. Bayesian Hierarchical Mechanism Design Using a Forest Matching Algorithm

To design a mechanism for CR networks, we need to develop the conflict-solving rule for operators and sub-bands and, for the SUs, we should establish the competition policies that can lead to the optimal Bayesian Nash equilibrium. This motivates us to model the interaction between the SUs and the PU network as a two-sided matching market. In the rest of this section, we propose a distributed algorithm that approaches a unique and stable matching structure which coincides with the optimal Bayesian Nash equilibrium of the DSA game. We then introduce a *Bayesian hierarchical mechanism* that can incentivize truth-telling by all SUs.

We first model the PU network as a two-layer forest as follows. The PU network consists of a forest structure with L trees, each of which corresponds to an operator and its sub-bands. More specifically, the roots of the forest represent the operators, and the leaves represent the operators' sub-bands and associated ability to share their sub-bands with SUs (i.e., payoffs and channel gains associated with sub-band sharing). We introduce a forest matching market, in which the set of SUs will be first partitioned into L sub-sets, each of which corresponds to a set of SUs matched to the same operator (root). Each SU can then request a sub-band (leaf) of its matched

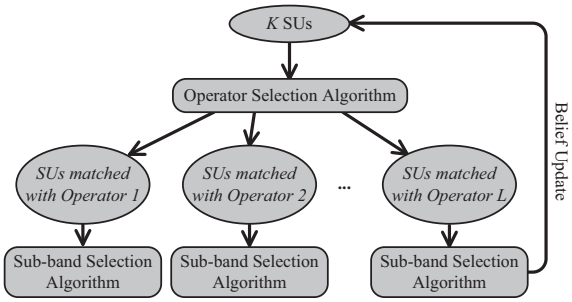

Fig. 3. The relationship between different algorithms for our stable forest matching market.

operator (root). Let the forest be $\mathcal{H} = \langle \mathcal{V}, \mathcal{E} \rangle$ where $\mathcal{V} = \mathcal{S} \cup \mathcal{O}$ is the set of vertices consisting of both roots and leaves and $\mathcal{E}$ is the set of edges connecting different vertices. In the PU network, only each root (e.g., operator i) and its corresponding leaves (e.g., sub-bands in $\mathcal{S}_i$) are connected with edges. We illustrate the forest matching market for the CR network of Figure 1 in Figure 2. Let us formally define the forest matching market as follows:

Definition 5: We define a forest matching market as $\mathcal{F} = \langle \mathcal{D}, \mathcal{H}, \succ \rangle$ where $\mathcal{D}$ is a set of agents, $\mathcal{H}$ is a forest structure, and $\succ$ is the preference.

We mainly focus on a (two-sided) forest matching market with a two-layer forest structure in one side of the market. In this market, the preference of each agent over a forest consists of two preference lists: the preference over the roots and the preference over the leaves. Similarly, each root or leaf can also have a preference over the agents. Our model can be extended into more general cases that contain a forest with more than two layers by introducing the preferences for agents (and each element in each layer) over elements in each layer (and agents), which will be discussed in Section VI. In the rest of this paper, we abuse the notation and use j and S^i_j to denote the jth root and the ith leaf in root j, respectively.

The above definition can be regarded as a generalization of the traditional two-sided stable matching markets [4], [11], [30] into a forest structure. Note that if there are no roots in the forest structure (called a zero-connection or zero-order forest [12]), the forest matching market becomes equivalent to the two-sided matching market.

Each agent $D_k \in \mathcal{D}$ can only obtain its payoff after being matched to a specific leaf belonging to a specific root. Let us define the matching between the agents and a forest as follows:

Definition 6: We define a (2-layer forest) matching as $M = \langle M^o, M^b \rangle$ where

1) M^o is a function from the set $\mathcal{D} \cup \mathcal{O}$ into the set of unordered families of elements of $\mathcal{D} \cup \mathcal{O}$ such that $|M^o(D_k)| = 1$ for every agent D_k and $M^o(D_k) = D_k$ if $M^o(D_k) \notin \mathcal{O}$, $|M^o(i)| \leq q_i$ for every $i \in \mathcal{O}$, and $M^o(D_k) = i$ if and only if $D_k \in M^o(i)$,
2) M^b is a function from the set $\mathcal{D} \cup \mathcal{S}$ onto itself such that if $M^b(D_k) \neq D_k$, then $M^b(D_k) \in \mathcal{S}$ and if $M^b(S^k_j) \neq S^k_j$ then $M^b(S^k_j) \in \mathcal{D}$, and

$$M^b(D_k) = S_j^k \Leftrightarrow M^b(S_j^k) = D_k \ \forall D_k \in M^o(j).$$

In our model, not all SUs will be accepted by the operators. We use $M^o(D_k) = D_k$ or $M^b(D_k) = D_k$ to mean that SU D_k cannot be matched with any operator or sub-band. Note that in the forest matching market, for each agent D_k, being matched with a root i cannot guarantee that it will also be matched with a leaf in $\mathcal{S}_i$, i.e., even if D_k satisfies $M^o(D_k) \neq D_k$, it can still have $M^b(D_k) = D_k$.

Let us define the dynamic spectrum access of SUs in a CR network as a forest matching market $\mathcal{F}^{DSA} = \langle \mathcal{D}, \mathcal{H}, \succ \rangle$ in which the agents are the SUs. $\mathcal{H}$ is the forest structure of the PU network, where the operators are the roots and the sub-bands licensed to the operators are the leaves.

Our main objective is to develop an algorithm that can achieve a stable matching between the SUs and the PU network. We introduce the concept of stability for the forest matching market as follows:

Definition 7: A (forest) matching M is said to be *stable* if every agent believes that matching M cannot be strictly improved upon by any agent, agent-and-root, or agent-and-leaf pair.

Finding the optimal action profile of SUs requires us to jointly optimize two sub-problems, the operator and sub-band selection sub-problems, and the belief function of SUs. In the remainder of this section, we first model the operator selection sub-problem as a *two-sided many-to-one matching market with private belief* (to be discussed in Section V-A) in which the SUs will be matched to L operators. We then model the sub-band selection sub-problem as a *two-sided one-to-one matching market with private belief* (to be discussed in Section V-B). At the end of each time slot, each SU obtains its payoff and updates its belief using a Bayesian belief updating algorithm (to be discussed in Section V-C). We illustrate the relationship between different markets and corresponding algorithms in Figure 3.

A. Operator Selection Sub-market

In this and next subsections, we assume that each SU D_k has a fixed private belief function $b_{D_k}(\boldsymbol{a}_{-D_k})$. We will relax this assumption and discuss the Bayesian belief updating algorithm in Section V-C.

Each SU D_k tries to be matched with an operator which is believed to be able to provide the sub-band that can maximize the payoff of D_k. We model this problem as a two-sided many-to-one matching market with private belief, which is defined as follows:

Definition 8: A (two-sided many-to-one) matching market with private belief is a market $\mathcal{G}^{M1} = \langle \mathcal{D}, \mathcal{O}, \boldsymbol{b}, \succ \rangle$ where $\mathcal{D}$ and $\mathcal{O}$ are two finite and disjoint sets of agents, $\boldsymbol{b}$ is the vector of the given belief functions, and $\succ$ is the preference of each agent.

We define the operator selection sub-problem as a (two-sided many-to-one) matching market with private belief, referred to as the *operator selection sub-market*, in which $\mathcal{D}$ is the set of SUs, $\mathcal{O}$ is the set of operators and $\boldsymbol{b}$ is the belief of SUs.

In the operator selection sub-market, each SU D_k will first send a request to the operator which, according to the belief function of D_k, will allocate the payoff-maximizing sub-band to D_k. However, the final sub-band that will be allocated by each operator to SU D_k is not only related to action a_{D_k} of D_k but also depends on the types and belief functions of other SUs which are unknown to SU D_k. Each SU D_k needs to estimate the expected payoff obtained from each operator i using its belief function, i.e., for the given belief function $b_{D_k}(\boldsymbol{a}_{-D_k})$ of SU D_k, the expected payoff of D_k when D_k chooses action $a^o_{D_k} = i$ is given by

$$\begin{aligned} &\hat{\varpi}_{D_k}\left(a^o_{D_k} = i, b_{D_k}(\boldsymbol{a}_{-D_k})\right) \qquad (7)\\ &= \max_{a^b_{D_k} \in \mathcal{S}_i \cup \{\emptyset\}} \sum_{\boldsymbol{a}_{-D_k} \in \boldsymbol{\mathcal{A}}_{-D_k}} b_{D_k}(\boldsymbol{a}_{-D_k})\, \varpi_{D_k}\left[M^b(D_k)\right], \end{aligned}$$

where $\varpi_{D_k}\left[M^b(D_k)\right]$ is given in (2). Each SU D_k can then establish the preference list $\mathcal{R}^o_{D_k}$ about the operators by ranking the above expected payoffs obtained from each operator in (7) from the highest to the lowest.

We use $D_k \succ_i D_n$ to denote that operator i prefers SU D_k to SU D_n, i.e., $\eta_i^{\min}[D_k] > \eta_i^{\min}[D_n]$, and use $i \succ_{D_k} m$ to denote that SU D_k prefers accessing the spectrum of operator i to that of operator m for $i \neq m$ and $i, m \in \mathcal{O}$, i.e., $\hat{\varpi}_{D_k}\left(a^o_{D_k} = i, b_{D_k}(\boldsymbol{a}_{-D_k})\right) > \hat{\varpi}_{D_k}\left(a^o_{D_k} = m, b_{D_k}(\boldsymbol{a}_{-D_k})\right)$.

In the operator selection sub-market, we seek a matching M^o between SUs and operators that is *optimal for SUs*, that is, there is no stable matching M'^o such that $M'^o(D_k) \succ_{D_k} M^o(D_k)$ or $M'^o(D_k) = M^o(D_k)$ for all $D_k \in \mathcal{D}$ with $M'^o(D_n) \succ_{D_n} M^o(D_n)$ for at least one $D_n \in \mathcal{D}$.

Note that the operator selection sub-market is equivalent to the traditional two-sided many-to-one matching market [30], also called the college admission market (or the hospitals and residents market), except that, in the latter market, there is no belief function for each SU. This makes it possible for us to apply the deferred-acceptance algorithm [5], [11] to achieve a unique and stable matching between SUs and operators.

Before we present the detailed algorithm, let us briefly discuss the timing structure of CR networks. At the beginning of each time slot, the SUs send a request signal to their preferred operators and wait for confirmation. If an SU receives the confirmation of acceptance from the requested operator, it will then compete with the other accepted SUs for the sub-bands of the operator. We will provide a more detailed discussion about the sub-band selection sub-problem in the next subsection. Let us now present the detailed operator selection algorithm as follows:

Algorithm 1: An Operator Selection Algorithm

Input: Each SU D_k establish a preference $\mathcal{R}^o_{D_k}$ using (7). Every operator i establishes a preference $\mathcal{R}_i$.
Output: a matching M^o.

1) *Initilization*: Every SU D_k sends a request signal to its most preferred operator $a^o_{D_k} = \tilde{1}_{D_k}$,
2) WHILE *every SU is on the waiting list of an operator and no operator will reject any SU*
   i) Each SU D_k that receives a rejection message from operator i removes operator i from its preference $\mathcal{R}^o_{D_k}$,

and then sends a request signal to the most preferred operator in the updated $\mathcal{R}^o_{D_k}$.

ii) Each operator i establishes a waiting list and keeps up to q_i most preferred SUs that send requesting signals to it into its waiting list and rejects the remaining requesting SUs.

ENDWHILE

3) Every operator sends accepting message to the set of SUs in its waiting list.

The above algorithm is a direct application of the modified deferred-acceptance algorithm for the two-sided matching market introduced in [30]. Note that if an SU has been rejected by all operators at the end of the above algorithm, it cannot access any sub-band of the operators.

From Algorithm 1, we can write the conflict-solving rule for the operator as follows: if the number of SUs who send request to operator i exceeds q_i, operator i will send rejection messages to all the SUs except for the q_i most preferred SUs that have send requests to operator i so far.

We can prove the following result about Algorithm 1.

Proposition 1: Algorithm 1 terminates in a unique and stable matching and the resulting matching M^o between operators and SUs is optimal for SUs.

Proof: See Appendix A. ∎

We have the following result about the complexity of Algorithm 1.

Proposition 2: The complexity of Algorithm 1 is $O(LK)$ in the worst case where L is the number of operators and K is the number of SUs[5].

Proof: See Appendix B. ∎

B. Sub-band Selection Sub-market

After all SUs have been matched to the operators, each SU can then decide which specific sub-band it can access. To solve this problem, we can model this problem as a two-sided one-to-one matching market with private belief, which is defined as follows:

Definition 9: Let us define the two-sided one-to-one matching game with private belief as $\mathcal{G}^{M2} = \langle \mathcal{U}_i, \mathcal{S}_i, \boldsymbol{b}, \succ \rangle$, which consists of two sets of finite and disjoint sets of agents $\mathcal{U}_i$ and $\mathcal{S}_i$, a vector of beliefs and the preference $\succ$.

We model the sub-band selection sub-problem as a two-sided one-to-one matching market with private belief, referred to as the *sub-band selection sub-market*, in which $\mathcal{U}_i = M^o(i)$ is the set of SUs matched to operator i by Algorithm 1 and $\mathcal{S}_i$ is the set of the existing sub-bands controlled by operator i. The belief function is defined in Section IV. In the sub-band selection sub-market, the SUs accepted by the same operator (e.g., operator i) compete for set $\mathcal{S}_i$ of sub-bands. Each SU can also establish an estimated version of its expected payoff obtained from each of its sub-band selection action as follows: supposing D_k has been accepted by operator i, we define the estimated payoff of SU D_k when D_k sends a request for sub-band S_i^l as

$$\begin{aligned}&\hat{\varpi}_{D_k}\left(a^b_{D_k} = S_i^l, b_{D_k}\left(\boldsymbol{a}_{-D_k}\right)\right) \\ &= \sum_{\boldsymbol{a}_{-D_k} \in \boldsymbol{\mathcal{A}}_{-D_k}} b_{D_k}\left(\boldsymbol{a}_{-D_k}\right) \varpi_{D_k}\left[M^b\left(D_k\right)\right].\end{aligned} \tag{8}$$

Each SU D_k accepted by operator i can then establish its preference $\mathcal{R}^b_{D_k}$ over sub-bands in $\mathcal{S}_i$ by ranking the estimated payoff in each sub-band of operator i in (8) from the highest to the lowest values. Operator i can also evaluate the preference $\mathcal{R}^b_{S_i^l}$ of each sub-band S_i^l over the accepted SUs using its received request signals sent by the SUs. We abuse the notation and use $S_i^l \succ_{D_k} S_i^m$ to denote that SU D_k prefers sub-band S_i^l over sub-band S_i^m, i.e., $\hat{\varpi}_{D_k}\left(a^b_{D_k} = S_i^l, b_{D_k}\left(\boldsymbol{a}_{-D_k}\right)\right) > \hat{\varpi}_{D_k}\left(a^b_{D_k} = S_i^m, b_{D_k}\left(\boldsymbol{a}_{-D_k}\right)\right)$. Similarly, $D_k \succ_{S_i^l} D_n$ means sub-band S_i^l prefers SU D_k over SU D_n, i.e., $\eta_{S_i^l}[D_k] > \eta_{S_i^l}[D_n]$ for $D_k, D_n \in M^o(i)$.

Similar to the operator selection sub-market, we seek a matching M^b between SUs and sub-bands of their matched operator that is *optimal for SUs*, that is, there is no stable matching M'^b between the set $M^o(i)$ of SUs and set $\mathcal{S}_i$ of sub-bands such that $M'^b(D_k) \succ_{D_k} M^b(D_k)$ or $M'^b(D_k) = M^b(D_k)$ for all $D_k \in M^o(i)$ with $M'^b(D_n) \succ_{D_n} M^b(D_n)$ for at least one $D_n \in M^o(i)$.

Similarly, we can observe that the above matching market is equivalent to the traditional two-sided one-to-one matching market [30], also called the stable marriage market, with the exception that set $\mathcal{U}_i$ and the preference of each SU depend on its belief. We can again apply the deferred-acceptance algorithm to achieve a unique, optimal and stable matching. We refer to this algorithm as **Algorithm 2: sub-band selection algorithm**. This algorithm is similar to the operator selection algorithm described in Algorithm 1 with the difference that the quota for each sub-band is 1. Similarly, we can write the conflict-solving rule for the sub-band as follows: if there are two or more SUs who send request for sub-band S_i^l, operator i will send rejection messages to all the SUs except for the most preferred SU that sends request for sub-band S_i^l so far. We omit the detailed description of the algorithm due to space limitations.

We have the following results.

Proposition 3: Algorithm 2 terminates in a unique and stable matching and the resulting matching M^b between SUs and sub-bands is optimal for SUs .

The proof of the above proposition follows the same line as that of Proposition 1. We omit the detailed derivation due to the space limitations.

We have the following result about the complexity of Algorithm 2.

Proposition 4: The complexity of Algorithm 2 for every operator i in the worst case is $O\left(N_i \cdot |\mathcal{U}_i|\right)$.

The proof of the above proposition follows the same line as that of Proposition 2. We omit the details.

C. Bayesian Hierarchical Mechanism

In this subsection, we introduce a Bayesian belief updating algorithm for all SUs to iteratively update their

[5]We use Bachmann-Landau notation: $f = O(g)$ if $\lim_{n\to\infty} \frac{f(n)}{g(n)} < +\infty$.

beliefs. We use $[t]$ to denote the parameters in the tth time slot. From Proposition 1 in Section III, it can be observed that for each action profile $\boldsymbol{a}$, belief function and preference of SUs, the resulting matching M^o between SUs and operators is also determined. From Proposition 3, we can observe that for every matching M^o achieved by Algorithm 1, the preference of each SU D_k over sub-bands of its matched operator is also fixed. Combining these two observations, we can claim that, for every given preference profile $\boldsymbol{\mathcal{R}}$ which is calculated by the beliefs of SUs, the resulting matchings in both operator and sub-band selection sub-markets using Algorithms 1 and 2 are fixed. To optimize the matching for the operator selection and sub-band selection sub-markets, SUs only need to determine their belief function. Following the same line as Section III, each SU's belief regarding the resulting matchings and the preferences of other SUs follow from an unknown stationary distribution [9], and hence each SU can use the following equation to calculate the belief about the operator selection action profile of other SUs at the beginning of each time slot t,

$$b_{D_k}\left(\boldsymbol{a}_{-D_k}[t]\right) = \frac{\theta_{D_k}\left(\boldsymbol{a}_{-D_k}[t-1]\right)}{t-1} \tag{9}$$

where $\theta_{D_k}\left(\boldsymbol{a}_{-D_k}[t-1]\right) = \sum_{u\in\{1,\ldots,t-1\}} \mathrm{Dir}\left(\boldsymbol{a}_{-D_k}[u] = \boldsymbol{a}_{-D_k}[t-1]\right)$ is the number of times that SU D_k observes actions $\boldsymbol{a}_{-D_k}[t-1]$ of other SUs during the previous $t-1$ time slots and $\mathrm{Dir}(\cdot)$ is the Dirac delta function. After updating its belief using (9), each SU updates its preference over operators and sub-bands using (7) and (8), respectively. The main idea of the above belief updating rules is that each SU estimates the resulting matching using the frequency with which each matching has been observed in the previous history.

Since each SU cannot have any observation history before the start of the spectrum access process, it is necessary for each SU to set a prior distribution $b_{D_k}\left(\boldsymbol{a}_{-D_k}[0]\right)$ at the beginning of the process. This prior can be obtained by allowing all SUs to go through a training process. More specifically, all SUs can randomly choose their operators to establish a prior distribution during the training period. Note that the prior distribution obtained by each SU does not affect the long-term learning process of SUs because as each SU receives more and more observations over time, the effects of this prior distribution will be outweighed [9].

Let us present the Bayesian hierarchical algorithm as follows:

Algorithm 3: A Bayesian Hierarchical Algorithm

Initialization: Each SU D_k has a prior belief $b_{D_k}\left(\boldsymbol{a}_{-D_k}[0]\right)$,
WHILE *the matching of the forest matching market is not stable*,

1) SUs enter the operator selection sub-market and apply Algorithm 1 to find the stable matching M^o,
2) After being matched to the operators, SUs enter the sub-band selection sub-market and apply Algorithm 2 to find the stable matching M^b,
3) After all SUs are matched to the operators and sub-bands, they use equation (9) to update their beliefs and then apply equations (7) and (8) to update their preferences about the operators and sub-bands at the beginning of the next time slot.

ENDWHILE

Theorem 1: We have the following results about Algorithm 3:

1) For the resulting beliefs of SUs, Algorithm 3 terminates in a unique and stable matching M^*, and the Bayesian hierarchical mechanism associated with Algorithm 3 is incentive compatible for SUs.
2) Suppose the belief of each SU converges to a stable probability distribution before time slot t and matching $M[t]$ satisfies $M[t] = M^*$ where M^* is the stable matching with the resulting belief. Then $M[\tau] = M^*$ for all $\tau > t$ using Algorithm 3.
3) The action profile $\boldsymbol{a}^*$ achieved by Algorithm 3 is the unique and optimal Bayesian Nash equilibrium of the DSA game with the resulting beliefs.

Proof: See Appendix C. ■

In the rest of this sub-section, we derive the worst case complexity of Algorithm 3 in each iteration.

Proposition 5: The complexity of Algorithm 3 in each iteration in the worst case is given by $O(L^2N^2K)$ for $N = \max_{i=\mathcal{O}}\{N_i\}$.

Proof: See Appendix D. ■

Note that, in practice, the number of operators in each specific local area is always limited, e.g., most countries only have three or four major telecommunication operators (e.g., there are 4 major mobile telecommunication operators that provide services to cover most of the population in in the United States.). Therefore, if we can regard L as a small fixed integer, the complexity of each iteration of Algorithm 3 in the worst case can be rewritten as $O(N^2K)$.

VI. Extensions and Future Works

Our proposed Bayesian hierarchical mechanism design and stable forest matching framework can be extended to more complex network systems. In this section, we describe how to extend our proposed framework into the case with PU networks consisting of more than two layers (to be discussed in Section VI-A) and the case with multiple PUs and SUs sharing the same sub-band (to be discussed in Section VI-B). We will also discuss the possible directions of our future work in Section VI-C.

A. Bayesian Hierarchical Mechanism Design for Systems with More Than Two Layers

Some practical networks can consist of a hierarchical structure with more than two layers. For example, the PU network can be a heterogenous network in which each operator possesses multiple co-located macro-cells, micro-cells, and/or femto-cells. Each cell consists of a base station that controls the sub-band allocation. In this case, if each SU tries to access a sub-band, it needs to first send the request to an operator and, once its request is accepted, send the request to a base station. The SUs can only access the sub-bands after being accepted by

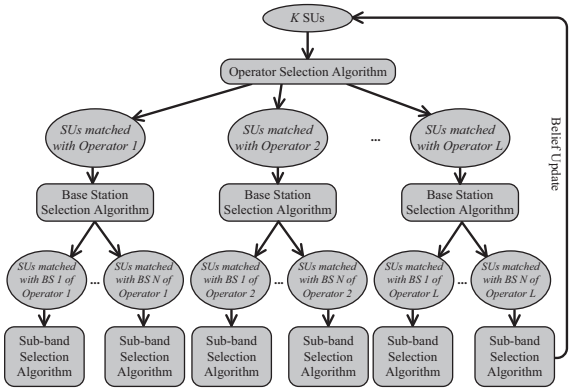

Fig. 4. A stable forest matching algorithm between SUs and the PU network with more than two layers.

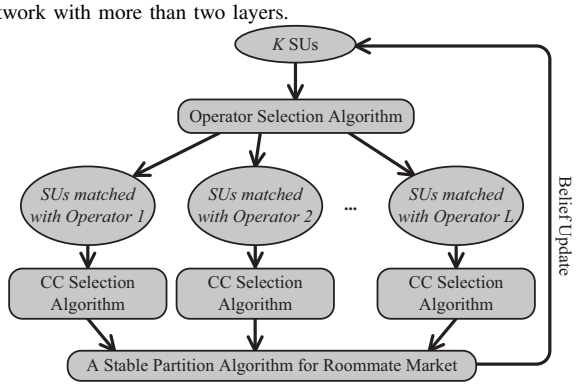

Fig. 5. A stable forest matching algorithm with multiple SUs sharing the same sub-band each of which consists of multiple component carriers (CC) selection algorithm.

both the requesting operator and base station. Since the SUs cannot exchange information with each other, we can again define the interactions of competing SUs as a Bayesian game. To design a distributed mechanism that can approach the optimal Bayesian Nash equilibrium, we can model the interaction between the SUs and the PU network as a forest matching market. Specifically, we can model the PU network as a 3-layer forest with operators as roots and each base station and its corresponding sub-bands as a branch. A unique and stable matching between the SUs and each layer of the forest structure can be achieved by the same two-sided matching algorithms as discussed in Sections V-A and V-B. Each SU will update its belief function after being matched with a sub-band. We illustrate the relationship between different matching algorithms for a 3-layer forest matching mechanism in Figure 4. Similarly, we can apply our Bayesian hierarchial mechanism design framework to optimize the system with a forest structure consisting of more layers.

B. Allowing Multiple SUs to Share the Same Sub-band

It can be observed that the spectrum utilization efficiency can be further improved by allowing multiple SUs to share the same sub-band. As mentioned previously, allowing multiple SUs to access the same sub-band requires careful design of the interference control rule for both SUs and PUs because even one SU with high transmit power can cause intolerable interference to all other SUs and PUs sharing the same sub-band. One way to support multi-SU sub-band sharing in a CR network is to impose a centralized interference control mechanism by the operators. More specifically, each operator i can allow more SUs to access its sub-bands, i.e., $q_i \geq N_i$, and keep monitoring the interference level for each of the sub-band sharing PUs and SUs. We define the set of sub-band sharing structures in each operator i as the set of all allocation schemes of q_i SUs to sub-bands in $\mathcal{S}_i$. Each operator will need to first evaluate the resulting revenues for all ${N_i}^{q_i}$ possible sub-band sharing structures and then choose the structure that can maximize its revenue. We can replace the conflict-solving rule defined in Section V with the above revenue-maximizing sub-band sharing rules for each operator. We can then apply the same belief updating methods in (9) for each SU to update its belief function after being matched with a sub-band and use equations (7) and (8) to decide its preferences over operators and sub-bands at the beginning of the next time slot.

Another way is to introduce a distributed coalition formation algorithm for the SUs to form different groups, each of which corresponds to a set of SUs sharing the same sub-band. More specifically, each SU after being accepted by operator i will not just establish a preference over all sub-bands in $\mathcal{S}_i$ but should establish a preference over all the possible sub-band sharing structures. SUs can then form different coalitions according to their preferences using distributed coalition formation algorithms [17], [40], [41]. Note that, different from Algorithm 2, in order for each SU to establish a preference over all the sub-band sharing structures, the SUs being matched with the same operator will need to coordinate and exchange information before accessing any sub-bands of the operator.

Our previously proposed hierarchical matching framework [42] can also be applied to enforce multiple SUs to share the same sub-band. To apply this framework, each operator will further divide each sub-band into multiple units, referred to as component carriers (CC). Each SU will be first allocated a CC using the same sub-band selection algorithm described in Algorithm 2 (sub-band in Algorithm 2 should be replaced by CC). After being allocated a CC, each SU can then decide whether to aggregate its allocated CC with the CCs of other SUs to further improve its performance. If multiple SUs agree to form a sub-band sharing pair, they will aggregate their CCs into a sub-band and share the sub-band with each other. We can then model the sub-band sharing problem as a roommate market where all SUs can be partitioned into groups using the stable partition/matching algorithm proposed in [43], [44]. We describe the relationship of different markets and corresponding algorithms of this method in Figure 5.

C. Future Works

From the previous discussion, it can be observed that our proposed stable forest matching algorithm is general and can be applied to more complex systems. Our results also point towards some new directions for future research. For example, in our model, we mainly focus on the distributed optimization of SUs and assume the conflict-solving rules of the operators are fixed. It has already been proved in [30], [45], for a two-sided matching market that, if all operators can know each other's preference as well as preferences of the SUs, they can adjust their conflict-solving rules to further improve their performance. Therefore, one future direction of our research is to study whether it is possible for the operators to also establish and maintain a belief function to further improve their expected revenues in a distributed fashion. Another potential direction for future work is to study the effect of allowing partial monetary transfers between PUs or SUs on the performance of CR networks [46], [47].

VII. Numerical Results

In this section, we present numerical results to assess the performance of our algorithms and mechanisms. Our proposed Bayesian hierarchical algorithm is general in the sense that each separate algorithm proposed for each of the sub-problems, i.e., the operator and sub-band selection sub-problems, can be individually applied to optimize CR networks under different conditions. More specifically, if SUs cannot establish a preference over the operators but are randomly matched to the available operators, they can still use the sub-band selection algorithm and the associated mechanism introduced in Section V-B to optimize their performance.

We consider a CR network in Figure 6 to simulate the interaction between the SUs and the PU network with a hierarchical structure. We model each SU as a transmission link (denoted as blue lines in Figure 6) from a source (denoted as a blue circle in Figure 6) to a destination (denoted as a green circle in Figure 6), and the PU network as a cellular system with a number of operators randomly located around the center of the coverage area (denoted as black rectangles in Figure 6) each of which consists of a fixed number of sub-bands and PUs (denoted as red triangles in Figure 6). In a practical system, a communication link between the source and destination should only be enabled when the source and destination are close enough. We hence assume the sources of the SUs are uniformly randomly located in the coverage area and each destination is uniformly randomly located within a fixed radius of its corresponding source. We also assume all PUs are uniformly randomly located in the coverage area. We focus on the uplink transmission for the PU network and each PU corresponds to a transmission link from the PU to the corresponding operator. We consider the payoff and revenue functions defined in (2) and (4) in Sections III and IV. Let the channel gain between the source and destination of SU D_k in sub-band S_i^l be $h_{D_k[S_i^l]} = \frac{\tilde{h}_{D_k[S_i^l]}}{\sqrt{d^{\xi}_{D_k[S_i^l]}}}$ where $\tilde{h}_{D_k[S_i^l]}$ is a fixed channel fading coefficient, d_{D_k} is the distance between the source and destination of SU D_k and ξ is the pathloss exponent. We also consider the channel gain between SU D_k and PU P_i^l to be $h_{D_kP_i^l} = \frac{\tilde{h}_{D_kP_i^l}}{\sqrt{d^{\xi}_{D_kP_i^l}}}$ where $\tilde{h}_{D_kP_i^l}$ is the channel fading coefficient and $d^{\xi}_{D_kP_i^l}$ is the distance between D_k and P_i^l. In the remainder of this section, we present numerical results to illustrate the performance of our proposed algorithm under different conditions. We mainly compare the following four algorithms:

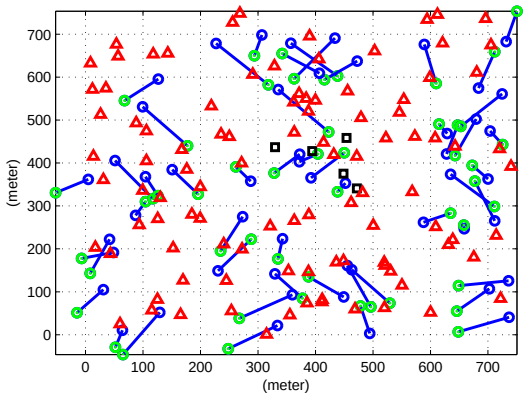

Fig. 6. Simulation setup: we simulate the PU network as a cellular network with multiple operators and the corresponding sub-bands. Each SU is a communication link from a transmitter to a receiver. We use □ to denote operators, ◊ to denote PUs and green colored and blue colored ○ to denote the transmitter and receiver of each SU, respectively.

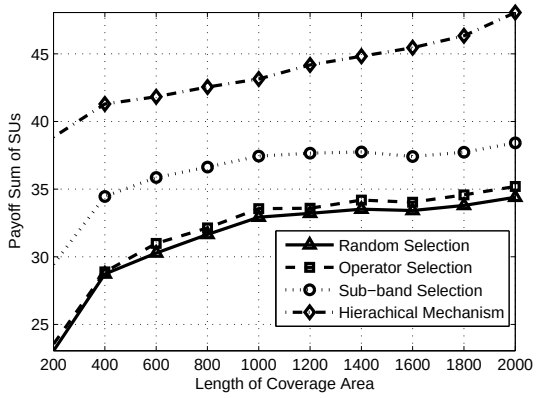

Fig. 7. The payoff sum of SUs for different sized networks with 50 SUs and 5 operators, each of which controls 10 sub-bands. Every sub-band contains a PU which is randomly located in the coverage area.

1) *Random Selection*: SUs are randomly matched to the operators and sub-bands.
2) *Operator Selection*: SUs are first matched to the operators using Algorithm 1 discussed in Section V-A. The SUs are then randomly matched to the sub-bands of their operators. This corresponds to the situation that each operator refuses to release all of

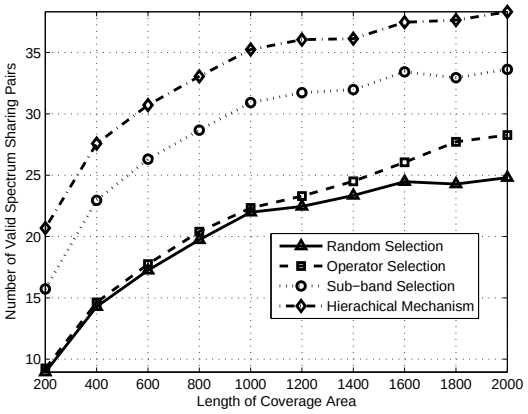

Fig. 8. The number of spectrum sharing pairs formed between SUs and PUs for different sized networks with 50 SUs and 5 operators, each of which controls 10 sub-bands. Every sub-band contains a PU which is randomly located in the coverage area.

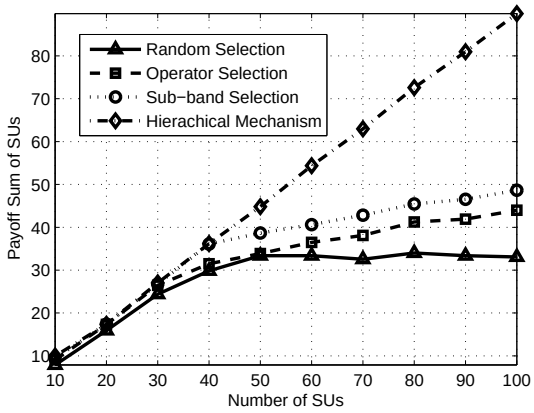

Fig. 9. The payoff sum of SUs under different numbers of SUs where the system consists of 5 operators, each of which controls 10 sub-bands. Every sub-band contains a PU which is randomly located in the coverage area.

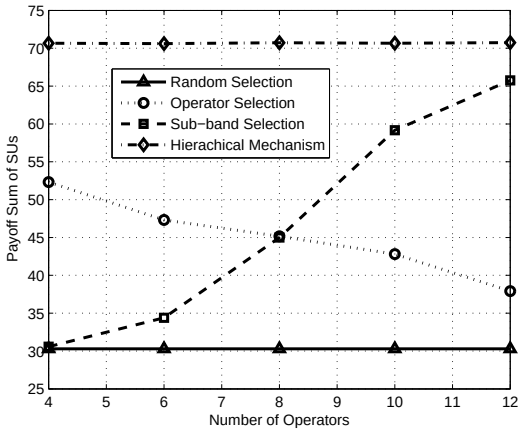

Fig. 10. The payoff sum of SUs under different numbers of operators. We consider a system with 120 PUs and 120 SUs, and each operator has $\frac{120}{L}$ PUs.

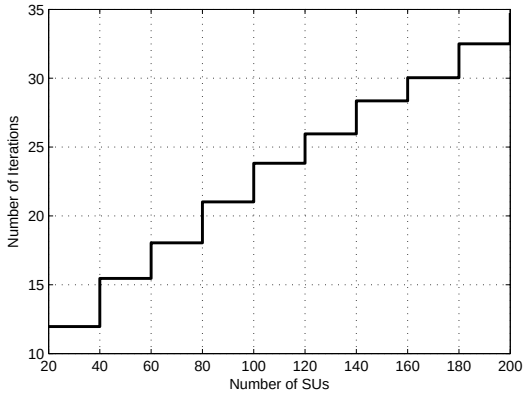

Fig. 11. The number of iterations required for all SUs to converge to the optimal Bayesian Nash equilibrium.

its sub-band information to SUs. In this case, each operator pre-selects a sub-band for each of the requesting SUs and only allows each SU to evaluate its payoff in its designated sub-band. Knowing the sub-band and the payoff that can obtain from the operators, each SU can then establish a preference over operators and then use Algorithm 1 to select its operator.

3) *Sub-band Selection*: SUs are first randomly matched to the operators. All SUs that are matched to the same operator will then try to be matched to the sub-bands using Algorithm 2 introduced in Section V-B. This may correspond to the case that the SUs cannot remember/store any previous observations about the sub-bands of the operators, i.e., a memoryless system.
4) *Hierarchical Mechanism*: SUs are matched to the operators and sub-bands by using the Bayesian hierarchical algorithm proposed in Section V-C.

Note that, as we have shown in Section V-C, if the SUs can update their belief functions using (9), the action profiles of SUs can always converge to the Bayesian Nash equilibrium for the resulting beliefs. In the rest of this section, we mainly focus on the case that SUs have already obtained their belief functions.

In Figure 7, we fix the number of operators and compare the payoff sum of SUs under different lengths of the square-shaped coverage area, with a range from 200 to 2000 meters. Our considered coverage area covers the femtocell, pico-cell ($<$ 200 meters), micro-cell ($>$ 200 meters) and macro-cell ($>$ 1000 meters) systems [48]. We observe that the random selection achieves the worst payoff among all mechanisms. We find that only limited payoff improvement can be achieved if each SU only applies the operator selection algorithm. This is because, in our simulation, the number of operators is limited and is much smaller than the number of sub-bands. Hence, the payoffs obtained by randomly selecting sub-bands in different operators are similar. However, if we apply sub-band selection algorithm for SUs to find their matchings, the payoff can be significantly improved. In other words, also optimizing the sub-band selection sub-problem among SUs provides much higher payoff

improvement than just optimizing the operator selection sub-problem. We can also observe that further performance improvement can be achieved by applying Bayesian hierarchical mechanism proposed in Section V-C.

In Figure 8, we consider the same setting as that of Figure 7 and assume that a spectrum sharing pair can only be formed between an SU and a PU if both of their payoffs and revenues exceed a fixed threshold. Again, we observe that, comparing to the random selection, the sub-band selection allows more spectrum sharing pairs to be formed between SUs and PUs.

In Figure 9, we compare the payoff of SUs for different numbers of SUs. We observe that if the number of SUs is small, allowing SUs to use our proposed hierarchical mechanism cannot provide much payoff improvement compared to the random selection. However, continuously increasing the number of SUs increases the competition among SUs for operators and sub-bands, and hence allowing SUs to use our proposed algorithm to optimize their sub-bands, or operators, or both can significantly improve their payoffs.

In Figure 10, we fix the number of SUs and PUs and compare the payoff sum of SUs under different numbers of operators. We observe that, by applying the operator selection algorithm, the payoff sum of SUs increases with the number of operators. However, the payoff sum achieved by sub-band selection decreases with the number of operators.

To study the convergence performance of our proposed hierarchical mechanism, we present the number of required iterations for SUs to approach the optimal Bayesian Nash equilibrium in Figure 11. We observe that the convergence performance of our proposed algorithms in many practical systems can be much better than the worst case convergence performance discussed in Section V-C. In many practical CR networks, different SUs have different relative distances to PUs and hence always result in different payoffs when accessing different sub-bands and operators. Only a limited number of SUs may choose the same preferred sub-band, and the chance of more than q_i SUs choosing the same operator i is also low, even when the number of SUs grows large. Therefore, our proposed mechanism has the potential to significantly improve the performance with a fast convergence rate in some practical systems.

VIII. Conclusion

In this paper, we study CR networks in which the PU network has a hierarchical structure consisting of a set of operators, each of which has been licensed a set of sub-bands. We model the dynamic spectrum access of SUs in this CR network as a Bayesian non-cooperative game, called DSA game. To develop a distributed mechanism for our proposed game, we propose a novel forest matching market to model the interaction between the SUs and the PU network. We divide the dynamic spectrum access problem for SUs into two sub-problems: the operator and sub-band selection sub-problems, and then propose operator and sub-band selection algorithms to optimize these sub-problems. We combine these algorithms with a Bayesian belief updating algorithm and propose a Bayesian hierarchical algorithm that can result in a unique and stable matching that coincides with the optimal Bayesian Nash equilibrium of our proposed DSA game. We prove that the Bayesian hierarchical mechanism associated with our proposed algorithm can incentivize true-telling by all SUs.

Appendix

A. Proof of Proposition 1

The proof of the above Proposition follows the same line as that in [30]. We provide a brief description of the proof for completeness. From Step 2) in Algorithm 1, we can easily show that if an SU D_k has been rejected by an operator i, there must exist at least q_i other SUs which are strictly preferred by operator i over SU D_k, and hence any matching between SU D_k and operator i must not be stable. Using this observation, we can also establish that if an SU D_k has been rejected by operator i, all the SUs that are less preferable to operator i than SU D_k will also be rejected by operator i.

Combining the above two observations, if q_i SUs and an operator i are matched at the end of Algorithm 1, we can claim that there is no other SU that is more preferred by operator i than the q_i SUs in the resulting matching structure. This is from the fact that if such an SU, say D_n, exists, at least one of the SUs in the final set of q_i SUs matched to operator i will be rejected by operator i in Algorithm 1. And, similarly, each SU matched to operator i cannot find another operator j that is more preferable than operator i in the resulting matching structure, because if such an operator j exists these SUs will not send a request message to operator i.

B. Proof of Proposition 2

In Algorithm 1, the worst case happens when all SUs can only choose the least preferred operator after receiving $(L-1)$ rejections from the operators. In this worst case, every SU will first send requests to $(L-1)$ most preferred operators and then receive rejections from all of them. In this case, the number of requests sent by K SUs is $K(L-1)$, which results in complexity of $O(KL)$.

C. Proof of Theorem 1

First, let us consider the first part of result 1). Combining Propositions 1 and 3, we can claim that for the given beliefs at SUs, the matchings resulted from both operator and sub-band selection algorithms are unique and stable. Since Step 1-2) in Algorithm 3 is equivalent to Algorithms 1 and 2, the matching achieved by Algorithm 3 is also unique and

stable for the resulting beliefs of SUs. We will present the proof of the second part of result 1) at the end of this proof.

We now consider result 2). If $M[t] = M^* = \langle M^{o*}, M^{b*} \rangle$ in time slot t, we then have $\bar{\varpi}_{D_k}\left(a^*_{D_k}, b_{D_k}\left(\boldsymbol{a}^*_{-D_k}\right)\right) > \bar{\varpi}_{D_k}\left(a'_{D_k}, b_{D_k}\left(\boldsymbol{a}_{-D_k}\right)\right)$ in time slot t. Let us show that in the next time slot $t+1$, each SU will stick with M^* and will not change to other actions. In time slot $t+1$, SU D_k will update its belief as follows:

$$b_{D_k}\left(\boldsymbol{a}_{-D_k}[t+1]\right) = \alpha b_{D_k}\left(\boldsymbol{a}_{-D_k}[t]\right) + (1-\alpha)\mathrm{Dir}\left(\boldsymbol{a}_{-D_k}[t+1]\right), \tag{10}$$

where $\alpha = \frac{t}{t+1}$. We can then rewrite the updated payoff function of D_k as

$$\begin{aligned}&\bar{\varpi}_{D_k}\left(a_{D_k}[t+1], b_{D_k}\left(\boldsymbol{a}_{-D_k}[t+1]\right)\right)\\ &= \alpha\bar{\varpi}_{D_k}\left(a_{D_k}[t], b_{D_k}\left(\boldsymbol{a}_{-D_k}[t]\right)\right) + \\ &\quad (1-\alpha)\bar{\varpi}_{D_k}\left(a_{D_k}[t+1], b_{D_k}\left(\boldsymbol{a}_{-D_k}[t+1]\right)\right),\end{aligned}$$

which is a linear combination of $\bar{\varpi}_{D_k}[t]$ and $\bar{\varpi}_{D_k}[t+1]$. It can be easily observed that choosing $a_{D_k}[t+1] = a_{D_k}[t] = a^*_{D_k}$ maximizes both payoff functions of SU D_k. This process is repeated in the following time slots.

Let us consider result 3). First, from the definition of stable matching in Definition 7, we can claim that for a given stable matching M^o or M^b, no SU has the intention to deviate from M^o or M^b by choosing another operator or sub-band. In addition, according to the definition of stable matching, if M^o is stable, there is no other matching M'^o such that D_k and D_n are matched to i and j, respectively, and also satisfies $j \succ_{D_k} i$ and $D_k \succ_j D_n$. In other words, if two SUs can switch their selected operators or sub-bands to improve their payoffs, they are not in a stable matching. However, they may still be in the Bayesian Nash equilibrium [10]. We hence can claim that, for both operator and sub-band selection sub-markets, the payoff sum of SUs achieved by the action profile of SUs in a stable matching equals or is greater than that achieved by the action profile in a Bayesian Nash equilibrium but not a stable matching.

We can also observe that Algorithms 1 and 2 are equivalent to a specific deferred-acceptance algorithm in which SUs send their requests for the operators and sub-bands first. This specific algorithm is also called a deferred-acceptance algorithm with SU proposing, which has the following property.

Proposition 6: If M^o and M^b are the resulting matchings of the deferred-acceptance algorithm with SU proposing for operator and sub-band selection sub-markets, then we have the following results: 1) For the operator selection sub-market, there is no other matching M'^o such that $M^o(D_k) \succeq_{D_k} M'^o(D_k)$ with $M^o(D_n) \succ_{D_n} M'^o(D_n)$ for at least one $D_n \in \mathcal{D}$, 2) For the sub-band selection sub-market, there is no other matching M'^b such that $M^b(D_k) \succeq_{D_k} M'^b(D_k)$ with $M^b(D_n) \succ_{D_n} M'^b(D_n)$ for at least one $D_n \in \mathcal{U}_i$ where $i = M^o(D_n)$.

From the above results, we can claim that the matching achieved by Algorithms 1 and 2 obtains the optimal Bayesian Nash equilibria.

Let us consider the second part of result 1). Using the above results, we can show that if each belief profile of SUs corresponds to a unique action profile, we can use the same method as in Proposition 1 to prove that there is no other action profile for SUs that will provide higher payoffs for SUs. In other words, misrepresenting the action for each SU cannot provide any improvement for its payoff. This concludes the proof.

D. Proof of Proposition 5

In each time slot, all SUs needs to go through Steps 1) to 3) in Algorithm 3, which contains Algorithm 1 with a complexity of $O(KL)$ and Algorithm 2 with a complexity of $\sum_{i\in\mathcal{O}} O\left(N_i|\mathcal{U}_i|\right)$. Using the fact that $N_i < N$ and $|\mathcal{U}_i| \leq N_i$, we can claim that each iteration of Algorithm 3 has a complexity of $O(KLLN^2)$.

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