Transmission Strategy with Cooperative Sensors in Cognitive Radio Networks

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***Abstract*—Cooperative spectrum sensing is a technology that allows cooperative sensors to assist cognitive radio (CR) transmitters to intelligently decide their transmission opportunities. Transmissions can only be successful if spectrum is available for both CR transmitters and receivers. Motivated by this observation, we use Boolean-Poisson model to analyze the geometric property of the geographical region which allows CR transmission to be helped with cooperative sensors. We find that cooperative sensing cannot always be helpful and the region allowing CR transmission is generally not circular symmetry. We identify the condition that transmission link is bidirectional. We further extend this model into the sub-optimal scenario among the secondary users and the corresponding transmission allowable region. We derive the condition for which secondary users cannot reduce their Bayesian risk by using cooperative sensors. We conclude with the guidelines for deploying CRs into the existing network.**

***Index Terms*—Cognitive radio, Bayesian risk, Nash equilibrium, Min-Max, Cooperative sensor**

I. Introduction

Cognitive radio (CR) allows each transmitter to first sense the availability of the spectrum and then intelligently decide its transmission actions based on the sensing results. It attracts significant interest due to its potential to improve the utilization of the spectrum [2], [3]. One solution to improve the accuracy of the sensing is to allow each CR transmitter to be helped by its nearby sensors, commonly refers to as the cooperative sensors [4]–[11].

Nevertheless, coordination and information exchange between CR transmitters and cooperative sensors also introduce communication overhead and increase power consumption. To reduce the overhead of feedback information, the authors in [12] quantize the feedback information to approach the performance of soft-decision based sensing. It has been shown that the hard decision fusion rules such as AND-rule [13], OR-rule [4] and counting-rule [14] can be applied to reduce the overhead of feedback information. Authors in [15] proposed a threshold-based sensing approach which can further reduce the communication overhead by efficiently combing the feedback information. It was observed that the information provided by some cooperative sensors may not always be accurate. Even it is accurate, it may not always provide enough contribution to the performance of CR networks compared to the adverse impact caused by the extra communication overhead. For this reason, choosing the proper cooperative sensors is an important issue [16]–[22]. Most existing works do not take into consideration of the location and distribution of the CR transmitters, receivers and cooperative sensors. In addition to the information about the existence of primary transmitters, we are interested in the geographical region in which CR transmission can be successful. Geographical distribution is crucial for spatial spectrum reuse and determining the topology of CR networks. In this paper, we are interested in the feasible region allowing transmission between secondary users. These geographical information can also help build up a spectrum map [23], [24] or opportunistic routing protocol [25] in CR networks.

In this work, we explore the necessary condition to use cooperative sensors from the perspective of heterogeneous spectrum availability at secondary transmitters and receivers. The diverse distribution of secondary communication links was discussed in [26] from the information theoretic point of view. It is shown that even though cooperative sensing can help transmitters recover the transmission link, it does not guarantee that receivers can also utilize this link. That is, we cannot assume bidirectional and symmetric property of a communication link [27]. [28] provides a two-dimensional sensing approach to solve the heterogeneous problem. Different from previous works that assumes only one primary transmitter in the network, we apply Poisson point process [29], [30] to model the spatial distribution of multiple primary and secondary users. Finding the globally optimal solution for large multi-user networks cannot always be possible especially when the central controller is not availably. In this paper, distributed optimization for the worst-case performance of the secondary users are investigated using tools from the game theory. We try to understand secondary users' best transmission performance under the guaranteed performance of primary users.

By comparing the geometric property of geographic region, we found that:

1) Cooperative sensing can bring benefit to the secondary users only when it is located in a specific region around transmitter.
2) Even though the secondary users do not know the density of secondary users about the network, the transmission region is the same as that the density information is known to each secondary user.

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3) There exists a tradeoff between the collision probability and successful transmission while using cooperative sensors in ad hoc CR network [31].

A. Related Work

Authors in [16]–[22] studied the cooperative sensor selection problem in CR networks. Generally speaking, there are two fundamental questions for cooperative sensor selection:

1) How many sensors should be used for each CR transmitter and receiver to properly discover the vacant spectrum.
2) How to choose cooperative sensors to help the decision making of each CR transmitter and receiver.

The first question has been considered in [16]–[18]. Specifically, [16] proposed an algorithm to minimize the number of cooperative sensors under certain detection error constraint. The reporting overhead is taken into account in [17]. The number of reporting cooperative sensors should satisfy the limited required reporting time. In [18], the optimal number of cooperative sensors is derived to maximize throughput. The second question has been considered in [19]–[22]. In [19], a cooperative sensor selection criterion has been proposed to minimize total energy consumption of spectrum sensing and feedback energy to a fusion center. [20] studied the case that the cooperative sensors is deployed in a cellular network and only those who have enough signal-to-interference-and-noise are selected to help the decision making for each CR transmitter and receiver. The similar idea is also applied in [21] to find the best set of cooperative sensors. In order to increase the diversity of cooperative sensors, [22] develops correlation-aware scheme to select proper cooperative sensors.

In this paper, we mainly focus on the second question (2). We consider a CR network consisting of multiple secondary users and multiple primary users located in the same region. Each cooperative sensor can detect the activity of its nearby primary users. The primary users detected by each cooperative sensor may not necessarily be close to all the secondary users. Therefore, we focus on establishing the relationship between the detection results of cooperative sensors and the decision making process of the secondary users. We develop the criterion of choosing proper cooperative sensors for each secondary user based on the spatial geometrical property of CR networks.

B. Organization of The Paper

The rest of the paper is organized as follows: Section II describes the details of our model. In Section III, we formulate the optimization problem under the homogeneous scenario and provide the sub-optimal solution. In Section IV, we consider the heterogeneous scenario, that is, each secondary transmitter cannot know global information such as the density of secondary transmitters. The min-max solution is provided. We discuss the engineering meaning of Bayesian risk in Section V. In Section VI, we illustrate the numerical result based on previous analysis. We draw our conclusion in Section VII.

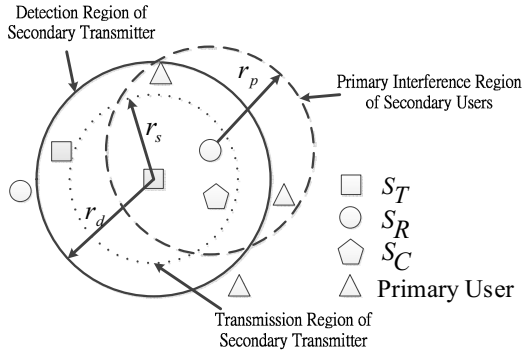

Fig. 1. Illustration of interference model. A communication link can exist if primary users are not in the detection region $\mathcal{B}(S_T, r_d)$ and interference protection region $\mathcal{B}(S_R, r_p)$, that is, $\mathbf{1}(S_T, r_d) = 1 \cap \mathbf{1}(S_R, r_p) = 1$.

II. System Model

A. Random Boolean-Poisson Model

Consider a CR network, as shown in Fig.1, in which a set of secondary links can access the spectrum licensed to a set of primary transmitters. Each secondary link corresponds to a communication channel from a secondary transmitter to a secondary receiver. Each secondary transmitter can either choose other idle secondary transmitters or receivers as its cooperative sensors within transmission region. Because secondary transmitters do not know whether the cooperative sensor can provide reliable information about existence of primary users nearby corresponding secondary receivers, it also needs to decide whether to believe the sensing result of the cooperative sensor.

We assume that all the spatial distribution of primary and secondary transmitters follow the homogeneous Poisson point process (PPP) Φ_p and Φ_s with density λ_p and λ_s respectively. The transmission of the secondary links and primary users is synchronized. This can be achieved by allowing all the secondary transmitters or receivers to eavesdrop on the synchronization signal sent by primary transmitters. The transmission radius of each secondary transmitter is denoted as r_s. To consider the worst case performance for receiver, we assume that the distance between secondary transmitter and receiver is r_s for each transmission pair[1] (we will relax this assumption in Section III). Equal distance is a common assumption while discussing wireless ad hoc network [32]–[34]. Through proper power control, the maximum radius which guarantees successful transmission is the same for each transmitter. Another way to interpret this assumption is that all the receivers are at the edge of the transmission range of their corresponding transmitters. This can help us to evaluate the worst-case performance of the system. By the stationary characteristic of homogeneous PPP [35], the statistics measured by the typical node at the origin is representative for all the other nodes. In the following, we choose a typical secondary transmitter (S_T), receiver (S_R), and a cooperative sensor (S_C) to illustrate our proposed optimization approach. Let $\mathcal{B}(S_T, r_d)$ be the detection region of S_T, where r_d is the radius

[1]Through discussing this scenario, we can find the minimal performance of the system.

of the detection region. That is, the secondary transmitters can successfully detect any active primary transmitters within $\mathcal{B}(S_T, r_d)$. Similarly, we can define the detection region for receiver S_R as $\mathcal{B}(S_R, r_p)$, where r_p is the radius of the propagation region of radio power from primary transmitter. S_R can successfully receive data sent by S_T if primary transmitter within $\mathcal{B}(S_R, r_p)$.

The list of notations used in this paper is provided in Table. I.

TABLE I
LIST OF NOTATIONS

Notation	Description
Φ_p	The set of primary transmitters
Φ_s	The set of secondary transmitters
λ_p	Density of primary transmitters
λ_s	Density of secondary transmitters
λ_a	Density of active secondary transmitters
r_p	Transmission radius of primary transmitters
r_s	Transmission radius of secondary transmitters
r_d	Detection radius of secondary transmitters
w, v	Collision cost with primary and secondary transmitters
p_t^o	Access probability of typical secondary transmitter
p_t	Access probability of all secondary transmitters except typical secondary transmitter
p_t^B	Access probability of all secondary transmitters at the Nash equilibrium without S_C in homogeneous case
p_t^{CB}	Access probability of all secondary transmitters at the Nash equilibrium with S_C in homogeneous case
p_t^{Het}	Access probability of all secondary transmitters at the Nash equilibrium without S_C in heterogeneous case
p_t^{CH}	Access probability all secondary transmitters at the Nash equilibrium with S_C in heterogeneous case
P_c	Probability of collision with secondary transmitters
α	Probability of existence of opportunistic link
β	Probability of existence of opportunistic link if S_C feedbacks "existence"
γ	Probability of non-existence of opportunistic link if S_C feedbacks "non-existence"
p	$\frac{\alpha\beta}{\alpha\beta+(1-\alpha)(1-\gamma)}$
q	$\frac{\alpha(1-\beta)}{\alpha(1-\beta)+(1-\alpha)\gamma}$

B. General Spectrum Sensing Model

In traditional spectrum sensing problem, a communication link can be established between S_T and S_R if they do not detect any active primary transmitters. To simplify the notation, we define indicator function $\mathbf{1}(A, r)$ as

$$\mathbf{1}(A, r) = \begin{cases} 1, & \text{if no primary user in } \mathcal{B}(A, r) \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

In Fig. 1, we can observe that the existence of opportunistic communication links depends on detection results for both secondary transmitter and receiver. Specifically, an opportunistic communication link exists if $\mathbf{1}(S_T, r_d) = 1 \cap \mathbf{1}(S_R, r_p) = 1$, and there is no opportunistic communication link if $\mathbf{1}(S_T, r_d) = 0 \cup \mathbf{1}(S_R, r_p) = 0$. Therefore, the existence of communication links $\mathbf{1}_{link}$ can be expressed as

$$\mathbf{1}_{link} = \mathbf{1}(S_T, r_d)\mathbf{1}(S_R, r_p), \tag{2}$$

$\mathbf{1}_{link} = 1$ means a communication link is available between secondary transmitter and receiver and $\mathbf{1}_{link} = 0$ means no such link between secondary transmitter and receiver. Therefore, secondary transmitters should on send its data if $\mathbf{1}(S_T, r_d) = 1$. We denote the probability of existing an opportunistic link between secondary transmitter and receiver given $\mathbf{1}(S_T, r_d) = 1$ as α, i.e. we can write

$$\alpha \triangleq \mathbb{P}(\mathbf{1}(S_R, r_p) = 1 | \mathbf{1}(S_T, r_d) = 1). \tag{3}$$

Spectrum sensing result obtained by S_C can be used to improve the accuracy of the detection results of S_T. By applying correlation between S_C and S_R, S_T can have more information about the receiver side. We use the following equations to express the relationship among S_T, S_R, and S_C. β and γ express the information about S_R in two different situations.

$$\begin{aligned} &\mathbb{P}(\mathbf{1}(S_C, r_d) = 1 | \mathbf{1}(S_T, r_d) = 1, \mathbf{1}(S_R, r_p) = 1) = \beta \\ &\mathbb{P}(\mathbf{1}(S_C, r_d) = 0 | \mathbf{1}(S_T, r_d) = 1, \mathbf{1}(S_R, r_p) = 0) = \gamma, \end{aligned} \tag{4}$$

β and γ are the probability of receiving $\mathbf{1}(S_C, r_d) = 1$ and $\mathbf{1}(S_C, r_d) = 0$ conditioned on $\mathbf{1}(S_C, r_p) = 1$ and $\mathbf{1}(S_R, r_p) = 0$.

III. HOMOGENEOUS CASE: NON-IDEAL SCENARIO ANALYSIS

A. Without Cooperative Sensor

There are two cases that can cause the failure to establish a communication link between a secondary transmitter and receiver. In the first one, the primary users do not exist in the detection region of the secondary transmitters but exist in the nearby region of the secondary receivers. In this case, since the secondary transmitters cannot detect the primary users, they can send the signals to the corresponding receiver. However, the secondary receivers cannot successfully, decode the signals because the interference caused by the primary users. In the second case, secondary transmitter erroneously detect the existence of the primary users and hence will not send any signals to the secondary receivers. However, secondary receivers do not detect any activity of the primary users in its detection region $\mathcal{B}(S_R, r_d)$ and hence still try to decode its received signals. In the first case, the transmission of each secondary transmitter will cause interference to both nearby primary users and other unintended secondary receivers. Since the main idea of the CR networks is to protect the primary users, we should assign a higher cost to the first case than to the second one. We define the Bayesian risk for each decision as follows:

$$\begin{aligned} R(T, p_t) &\triangleq \alpha v P_c + (1-\alpha) w \\ R(NT, p_t) &\triangleq \alpha(1 - P_c), \end{aligned} \tag{5}$$

where v is the penalty for collision with other secondary receivers and P_c is the resulting probability of collision with

other secondary receivers when spectrum access probability p_t given that the transmission of the secondary users does not conflict with other primary users. Since in this paper, we assume that the transmission of each secondary transmitter will cause to all the other secondary receivers and primary users within its transmission range defined by radius r_s, P_c is equivalent to the probability that the distance between the secondary transmitter and the nearest unintended secondary receiver is less than or equal to the distance r_s, i.e., we can write P_c as follows:

$$P_c = \mathbb{P}(|\mathcal{B}(S_R, r_s)|_{SU} \neq 0 | \mathbf{1}(S_T, r_d) = 1), \tag{6}$$

where $|.|_{SU}$ denotes the number of secondary transmitters in $\mathcal{B}(S_R, r_s)$. We normalize the risk for false-alarm to be 1 and let w and v be the risks for collision with primary users and secondary users, respectively. In the most ideal case, all the secondary transmitters try to minimize the Bayesian risk function defined as

$$R^g(p_t) \triangleq p_t R(T, p_t) + (1 - p_t) R(NT, p_t).$$

If a centralized controlled system, all the secondary transmitters can feedback the local information to a fusion center which will decide the best p_t for each secondary user. In this paper, we consider a distributed system in which secondary users cannot coordinate with each other but determine their own spectrum access probability in a distributed fashion.

Suppose the access probability conditioned on $\mathbf{1}(S_T, r_d) = 1$ is given by p_t^o. We can write Bayesian risk of secondary transmitter as

$$R(p_t^o, P_c) \triangleq p_t^o R(T, p_t) + (1 - p_t^o) R(NT, p_t). \tag{7}$$

We can observe that S_T cannot achieve the minimum risk function by only considering its own spectrum access probability p_t^o but should also take into consideration other secondary transmitters' spectrum access probability p_t. To analyze the results of the interactions among secondary users, we model the decision making process of the secondary users as a game. In this game, the secondary users are the players and the strategy of each player is to decide its spectrum access probability. We are interested at the solution commonly refereed to as the Nash equilibrium (NE) [36]. We formally define the NE of our proposed game as follows:

Definition 1: *Spectrum access probability $p_t^o \in [0, 1]$ and $p_t \in [0, 1]$ at the NE for typical secondary transmitter S_T is defined as*

$$\begin{aligned} p_t^B &= arg \min_{p_t^o \in [0,1]} R(p_t^o, \mathbf{p}_t^B) \\ &= arg \min_{p_t^o \in [0,1]} R(p_t^o, P_c^B), \end{aligned} \tag{8}$$

where $\mathbf{p}_t^B$ is the spectrum access probability of other secondary transmitter and the second equality comes from the fact that P_c is the function of $\mathbf{p}_t^B$. P_c^B is the resulting probability of collision with secondary transmitters at the NE.

Following Definition 1, we have the following results about the access probability p_t^o at the NE.

Proposition 1: *Suppose the NE has been reached in our proposed game. The spectrum access probability p_t^B is given by $p_t^B = \frac{\exp(\lambda_p \pi r_d^2)}{\pi r_s^2 \lambda_s} \ln \frac{\alpha(v+1)}{\alpha v + (1-\alpha) w}$.*

Proof: Note that by homogeneity of Φ_s the probability collision among secondary transmitters only depends on the distance between two secondary transmitters [37]. Therefore, the collisions occurs if there exists another secondary transmitter (which do not detect primary users) located within the distance smaller r_s around S_R we can express $\lambda_a = \lambda_s e^{-\lambda_p \pi r_d^2} p_t$ and

$$P_c \simeq 1 - e^{-\lambda_a \pi r_s^2}. \tag{9}$$

Eq. (9) follows from the fact that all the secondary transmitters are uniformly distributed on the planar area, the number of other secondary transmitters follows Poisson distribution.

One way to find the NE shown in *Definition 1* is that we can make all the decision having the same Bayesian risk function, that is, equal risk method in [38]. The Bayesian risk functions conditioned on T and NT is

$$\begin{aligned} R(T, P_c^B) &= \alpha v P_c^B + (1 - \alpha) w \\ R(NT, P_c^B) &= \alpha (1 - P_c^B). \end{aligned} \tag{10}$$

To arrange the equality above, we get

$$P_c^B = \frac{\alpha - (1 - \alpha) w}{\alpha (v + 1)}. \tag{11}$$

Then substitute eq.(9) into it, we get

$$p_t^B = \begin{cases} \dfrac{e^{\lambda_p \pi r_d^2}}{\pi r_s^2 \lambda_s} \ln \dfrac{\alpha(v+1)}{\alpha v + (1-\alpha) w}, & \textit{if } \alpha \geq \dfrac{w}{w+1} \\ 0, & \textit{if } \alpha < \dfrac{w}{w+1}. \end{cases} \tag{12}$$

■

Note that the value of the access probability needs to be between 0 and 1. If $p_t^B > 1$ (or < 0), it means that $R(T, p_t = 1) < R(NT, p_t = 1)$ (or $R(T, p_t = 0) > R(NT, p_t = 0)$).

B. With Cooperative Sensor

Now, we consider a homogeneous case that each secondary transmitter has been assigned with a cooperative sensor respectively. More specifically, in homogeneous case, all secondary transmitters learn the same values of α, β, γ. To simplify the notation, we define

$$p \triangleq \mathbb{P}(\mathbf{1}(S_R, r_p) = 1 | \mathbf{1}(S_C, r_d) = 1) = \frac{\alpha\beta}{\alpha\beta + (1-\alpha)(1-\gamma)}, \tag{13}$$

$$q \triangleq \mathbb{P}(\mathbf{1}(S_R, r_p) = 1 | \mathbf{1}(S_C, r_d) = 0) = \frac{\alpha(1-\beta)}{\alpha(1-\beta) + (1-\alpha)\gamma}, \tag{14}$$

respectively.

Now, the situation is slightly different from previous case because typical secondary transmitter S_T does not know the information obtained by other secondary transmitter from their cooperative sensors. The secondary transmitter S_T cannot know the Bayesian risk function of other secondary users. We denote the access probability of all the secondary transmitters except S_T as $p_{t|\mathbf{1}(S_T, r_d)=1}$ and $p_{t|\mathbf{1}(S_T, r_d)=0}$, and the resulting

$$R(p^o_{t|\mathbf{1}(S_C,r_d)=1}, P_c|\mathbf{1}(S_C,r_d)=1) \triangleq \\ p^o_{t|\mathbf{1}(S_C,r_d)=1} R(T,P_c|\mathbf{1}(S_C,r_d)=1) + (1-p^o_{t|\mathbf{1}(S_C,r_d)=1}) R(NT,P_c|\mathbf{1}(S_C,r_d)=1) \tag{15}$$

$$R(p^o_{t|\mathbf{1}(S_C,r_d)=0}, P_c|\mathbf{1}(S_C,r_d)=0) \triangleq \\ p^o_{t|\mathbf{1}(S_C,r_d)=0} R(T,P_c|\mathbf{1}(S_C,r_d)=0) + (1-p^o_{t|\mathbf{1}(S_C,r_d)=0}) R(NT,P_c|\mathbf{1}(S_C,r_d)=0) \tag{16}$$

collision probability is P_C. Then the Bayesian risk functions of T and NT conditioned on $\mathbf{1}(S_C,r_d)=0$ are given by,

$$R(T,P_c|\mathbf{1}(S_C,r_d)=0) = w(1-q)+qvP_c, \tag{17}$$
$$R(NT,P_c|\mathbf{1}(S_C,r_d)=0) = q(1-P_c). \tag{18}$$

Similarly, we can write the Bayesian risk functions conditioned on $\mathbf{1}(S_C,r_d)=1$ as

$$R(T,P_c|\mathbf{1}(S_C,r_d)=1) = w(1-p)+pvP_c, \tag{19}$$
$$R(NT,P_c|\mathbf{1}(S_C,r_d)=1) = p(1-P_c). \tag{20}$$

If we consider the spectrum access probability $p^o_{t|\mathbf{1}(S_C,r_d)=k}$ of S_T, the Bayesian risk function at each feedback information of S_T is shown in eq.(15) and (16) at the top of the page. From eq.(15) and (16), S_T needs to know P_c to determine the best access probability. However, P_c is closely related to the access probability of other secondary transmitters. Therefore, secondary transmitters need to modify access probability based on their beliefs about other secondary users' feedback information to minimize its own Bayesian risk function. We try to seek the NE in such scenario.

Definition 2: *The access probability at the NE conditioned on $\mathbf{1}(S_C,r_d)=k, k\in\{0,1\}$ for secondary transmitters is*

$$p^{CB}_{t|\mathbf{1}(S_C,r_d)=k} = \\ arg \min_{p^o_{t|\mathbf{1}(S_C,r_d)=k}\in[0,1]} R(p^o_{t|\mathbf{1}(S_C,r_d)=k}, P^{CB}_c|\mathbf{1}(S_C,r_d)=k), \tag{21}$$

where $k\in\{0,1\}$ and P^{CB}_c is the resulting probability of collision with other secondary transmitters at NE with spectrum access probability $p_{t|\mathbf{1}(S_C,r_d)=k} = p^{CB}_{t|\mathbf{1}(S_C,r_d)=k}$.

Then we have the following result.

Proposition 2: *We denote the access probability conditioned on $\mathbf{1}(S_C,r_d)=1$ and $\mathbf{1}(S_C,r_d)=0$ respectively as $p^{CB}_{t|\mathbf{1}(S_C,r_d)=1}$ and $p^{CB}_{t|\mathbf{1}(S_C,r_d)=0}$. Then the access probability in eq.(19)−eq.(18) at the NE are given in eq.(22) and eq.(23) at the top of the next page.*

Proof: Because all the cooperative sensors only sense the environment and feedback its own information to its own secondary transmitter, the sensing results are independent except those are close to each other. Therefore, we assume that all the sensing results are identical independent distribution (i.i.d.) and hence the active density λ_a can be expressed as

$$\lambda_a \simeq \lambda_s e^{-\lambda_p \pi r_d^2} \left(\mathbb{P}(\mathbf{1}(S_C,r_d)=1) p_{t|\mathbf{1}(S_C,r_d)=1} \right. \\ \left. + \mathbb{P}(\mathbf{1}(S_C,r_d)=0) p_{t|\mathbf{1}(S_C,r_d)=0}\right). \tag{24}$$

We first discuss about the case that $\mathbf{1}(S_C,r_d)=1$ and denote the P_c at the NE as P^{CB}_c. Following the same line as previous section, we get

$$R(T,P^{CB}_c|\mathbf{1}(S_C,r_d)=1) = R(NT,P^{CB}_c|\mathbf{1}(S_C,r_d)=1). \tag{25}$$

We hence can write the value of P^{CB}_c as

$$P^{CB}_c = 1-e^{-\lambda_a \pi r_s^2} = \frac{p-w(1-p)}{p(v+1)}, \ \textit{if } p \geq \frac{w}{w+1}. \tag{26}$$

The spectrum access probability at the NE should satisfy the above equation to guarantee that no secondary user intend to unilaterally deviate from its spectrum access probability conditioned on $\mathbf{1}(S_C,r_d)=1$. Combining eq.(9) and (24), we can observe that there are infinite values of P_c that can satisfy eq.(26). Considering the Bayesian risk function cannot keep equality conditioned on $\mathbf{1}(S_C,r_d)=0$. Therefore, we can determine the only one pair of solution because all secondary transmitters will always use T or NT conditioned on $\mathbf{1}(S_C,r_d)=0$. Substitute eq.(26) into eq.(17) and eq.(18)

$$R(T,P^{CB}_c|\mathbf{1}(S_C,r_d)=0) = (1-q)w + qv\frac{p-w(1-p)}{p(v+1)} \tag{27}$$
$$R(NT,P^{CB}_c|\mathbf{1}(S_C,r_d)=0) = q\left(1-\frac{p-w(1-p)}{p(v+1)}\right) \tag{28}$$

Secondary users always choose their transmission strategy that can minimize their Bayesian risk, that is

$$R\left(T,P^{CB}_c|\mathbf{1}(S_C,r_d)=0\right) \underset{T}{\overset{NT}{\gtrless}} R(NT,P^{CB}_c|\mathbf{1}(S_C,r_d)=0). \tag{29}$$

From the above equation, we obtain

$$p \underset{T}{\overset{NT}{\gtrless}} q. \tag{30}$$

Because it can be shown that p always larger than q, we can conclude $p^{CB}_{t|\mathbf{1}(S_C,r_d)=0} = 0$. Then the active density of secondary transmitters is

$$\lambda_a = \lambda_s e^{-\lambda_p \pi r_d^2} \mathbb{P}(\mathbf{1}(S_C,r_d)=1) p^{CB}_{t|\mathbf{1}(S_C,r_d)=1} \tag{31}$$

Using eq.(9), we can express $p^{CB}_{t|\mathbf{1}(S_C,r_d)=1}$ as

$$p^{CB}_{t|\mathbf{1}(S_C,r_d)=1} \\ = \frac{e^{\lambda_p \pi r_d^2}}{\lambda_s \pi r_s^2(\alpha\beta+(1-\alpha)(1-\gamma))} \ln \frac{p(v+1)}{pv+w(1-p)}, \tag{32}$$

where $\alpha\beta+(1-\alpha)(1-\gamma) = \mathbb{P}(\mathbf{1}(S_C,r_d)=1)$.

When the value of q is large enough, the access probability conditioned on $\mathbf{1}(S_C,r_d)=0$ should be larger than zero. To

$$p^{CB}_{t|\mathbf{1}(S_C,r_d)=1} = \begin{cases} 1, & \text{if } q \geq \dfrac{w}{w+1} \\ \dfrac{e^{\lambda_p \pi r_d^2}}{\lambda_s \pi r_s^2 \mathbb{P}(\mathbf{1}(S_C,r_d)=1)} \ln \dfrac{p(v+1)}{pv+w(1-p)}, & \text{if } q < \dfrac{w}{w+1},\ p \geq \dfrac{w}{w+1} \end{cases} \tag{22}$$

$$p^{CB}_{t|\mathbf{1}(S_C,r_d)=0} = \begin{cases} \dfrac{\frac{e^{\lambda_p \pi r_d^2}}{\lambda_s \pi r_s^2} \ln \frac{q(v+1)}{qv+w(1-q)} - \mathbb{P}(\mathbf{1}(S_C,r_d)=1)}{\mathbb{P}(\mathbf{1}(S_C,r_d)=0)}, & \text{if } q \geq \dfrac{w}{w+1} \\ 0, & \text{if } q < \dfrac{w}{w+1},\ p \geq \dfrac{w}{w+1} \end{cases} \tag{23}$$

find the NE in this situation, we can set $R(T, P_c|\mathbf{1}(S_C, r_d) = 0) = R(NT, P_c|\mathbf{1}(S_C, r_d) = 0)$ and write P_c at the NE is

$$P_c^{CB} = 1 - e^{-\lambda_a \pi r_s^2} = \frac{q - w(1-q)}{q(v+1)}, \text{if } q \geq \frac{w}{w+1}. \tag{33}$$

Following the similar steps in eq.(27)-eq.(29), we can get

$$q \underset{T}{\overset{NT}{\gtrless}} p \tag{34}$$

Because p is always larger than q, the access probability at the NE in this case is $p^{CB}_{t|\mathbf{1}(S_C,r_d)=1} = 1$. Then the active density of secondary transmitters is

$$\lambda_a = \lambda_s e^{-\lambda_p \pi r_d^2} \times \left(\mathbb{P}(\mathbf{1}(S_C,r_d)=1) + \mathbb{P}(\mathbf{1}(S_C,r_d)=0) p^{CB}_{t|\mathbf{1}(S_C,r_d)=0}\right). \tag{35}$$

Substituting the above equation above into eq.(33), we get

$$\mathbb{P}(\mathbf{1}(S_C,r_d)=1) + \mathbb{P}(\mathbf{1}(S_C,r_d)=0) p^{CB}_{t|\mathbf{1}(S_C,r_d)=0} = \frac{e^{\lambda_p \pi r_d^2}}{\lambda_s \pi r_s^2} \ln \frac{q(v+1)}{qv+w(1-q)}. \tag{36}$$

From eq.(26) and eq.(33), we can find that these two equations are can only satisfied if $p \geq \frac{w}{w+1}$ and $q \geq \frac{w}{w+1}$ respectively. Therefore, we can conclude that

$$P_c^{CB} = \begin{cases} \dfrac{q - w(1-q)}{q(v+1)}, & \text{if } q \geq \dfrac{w}{w+1} \\ \dfrac{p - w(1-p)}{p(v+1)}, & \text{if } q < \dfrac{w}{w+1}, p \geq \dfrac{w}{w+1}. \end{cases} \tag{37}$$

The corresponding access probabilities are expressed in eq.(22) and eq.(23), respectively. ■

C. Necessity of Cooperative Sensor

It has been observed that in CR networks, always trusting the result of the cooperative sensor may not result in good decision. For example, when the value of α is high, the communication link is reliable. S_T can ignore the information from cooperative sensor and always make the correct decision to transmit when $\mathbf{1}(S_T, r_d) = 1$. We can turn off the cooperative sensor to reduce the overhead of CR networks in this case. Therefore, we now focus on the condition for which cooperative sensor can minimize the Bayesian risk and when secondary transmitters can ignore the feedback information from cooperative sensor.

We introduce the following definitions to simplify notations in the following discussion.

Definition 3: *A cooperative sensor S_C is **unnecessary** if the S_T accesses channel with the same probability no matter what kind of information S_C feedbacks, i.e.:*

$$p_t^B = p^{CB}_{t|\mathbf{1}(S_C,r_d)=k} \neq 0, \text{ for } k = 0, 1.$$

Definition 4: *A cooperative sensor S_C is **necessary** if the S_T accesses channel with different probability according to what kind of information S_C feedbacks, i.e.:*

$$p^{CB}_{t|\mathbf{1}(S_C,r_d)=1} \neq p^{CB}_{t|\mathbf{1}(S_C,r_d)=0}.$$

Definition 5: *A cooperative sensor S_C is **useless** if the S_T always cannot access channel no matter what kind of information S_C feedbacks, i.e.:*

$$p_t^B = p^{CB}_{t|\mathbf{1}(S_C,r_d)=k} = 0, \text{ for } k = 0, 1.$$

Definition 6: *A **transmission allowable** region is the region where the probability of accessing channel of S_T is not 0, that is, $p_t^B \neq 0$ and $p^{CB}_{t|\mathbf{1}(S_C,r_d)=1} \neq 0$ for scenario with and without the cooperative sensor respectively.*

These four definitions explains the behavior of S_C according to its own different location relative to S_T. We give conditions for which the cooperative sensors can bring benefit to secondary transmitters or not under homogeneous and heterogeneous case as follows:

Theorem 1: *Cooperative sensor is unnecessary for a secondary transmitter if r_s satisfies*

$$r_s^2 \leq \frac{\exp(\lambda_p \pi r_d^2)}{\lambda_s \pi} \ln \frac{q(v+1)}{w(1-q)+qv}, \tag{38}$$

and is useless if

$$\alpha < \frac{w(1-\gamma)}{\beta + w(1-\gamma)} \tag{39}$$

and cooperative sensor is necessary if both eq.(38) and eq.(39) are not satisfied.

Proof: Due to lack of closed-form for *necessary* scenario, we first discuss about when cooperative sensor is *unnecessary* and *useless*. According to eq.(22) and eq.(23) in *Proposition* 2, we know that S_T always takes the same access probability for both of $\mathbf{1}(S_C, r_d) = 1$ and $\mathbf{1}(S_C, r_d) = 0$ only while $p^{CB}_{t|\mathbf{1}(S_C,r_d)=0} = 1$. Because r_s should be small enough to make sure that $p^{CB}_{t|\mathbf{1}(S_C,r_d)=0} = 1$, we substitute $p^{CB}_{t|\mathbf{1}(S_C,r_d)=0} = 1$ into eq.(36) and get eq. 38

$$r_s^2 \leq \frac{e^{\lambda_p \pi r_d^2}}{\lambda_s \pi} \ln \frac{q(v+1)}{qv+w(1-q)}.$$

For the *useless* region, S_T always decides to not access channel, that is $p^{CB}_{t|\mathbf{1}(S_C,r_d)=1} = 0$ and $p^{CB}_{t|\mathbf{1}(S_C,r_d)=0} = 0$. From eq.(22) and eq.(23), we can find that $p^{CB}_{t|\mathbf{1}(S_C,r_d)=1} = 0$ and $p^{CB}_{t|\mathbf{1}(S_C,r_d)=0} = 0$ if $\frac{p(v+1)}{pv+w(1-p)} \leq 1$ or $p < \frac{w}{w+1}$. Because $p = \frac{\alpha\beta}{\alpha\beta+(1-\alpha)(1-\gamma)}$, we can directly obtain that $\alpha < \frac{w(1-\gamma)}{\beta+w(1-\gamma)}$. ■

By applying **Theorem 1**, S_T can turn off its cooperative sensor to reduce the communication overhead if S_C cannot bring any useful information for S_T.

IV. Heterogeneous Case: Min-Max Analysis

In Section III, we study the case that all the secondary receivers and cooperative sensors have the same distance with the corresponding secondary transmitter. Secondary transmitters also know the density of secondary transmitters. With the help of this information, each secondary transmitter can calculate its access probability at the NE. We now consider a more realistic scenario in which the observation obtained by cooperative sensor from the environment is not homogeneous. In some practical systems, the global information such as the density of secondary transmitters is hard to obtain for secondary transmitters. Without this information, secondary users cannot know the Bayesian risk function or calculate the corresponding access probability to reach the NE. In this section, we consider the system in which secondary users try to minimize its own maximum Bayesian risk which is equivalent to the min-max criterion. In this section, we investigate the access probability for each secondary transmitter under min-max criterion.

A. Without Cooperative Sensor

We assume that all the other secondary transmitters take access probability p_t and the typical secondary transmitter S_T takes p^o_t. Without the global information about the secondary users. Each secondary user needs to consider the worst case of the collision probability.

Definition 7: *The min-max access probability p^{Het}_t without the cooperative sensor is*

$$p^{Het}_t = arg \min_{p^o_t \in [0,1]} \max_{P_c \in [0,1]} R(p^o_t, P_c). \tag{40}$$

The above results can be directly obtained from *Proposition* 3.

Proposition 3: *Based on the min-max criterion, the access probability p^{Het}_t without S_C satisfies*

$$p^{Het}_t = \begin{cases} \dfrac{1}{v+1}, & \text{if } \alpha \geq \dfrac{w}{w+1} \\ 0, & \text{if } \alpha < \dfrac{w}{w+1}. \end{cases} \tag{41}$$

Proof: Because the access probability of S_T is p^o_t, then we can rewrite Bayesian risk function as

$$\begin{aligned} R(p^o_t, P_c) &= p^o_t R(T, P_c) + (1-p^o_t) R(NT, P_c) \\ &= p^o_t(1-\alpha)w + (1-p^o_t)\alpha + P_c\alpha(p^o_t(v+1)-1). \end{aligned} \tag{42}$$

From eq.(42), we can find that $p^o_t(v+1)-1$ determines the maximum value of $R(p^o_t, P_c)$. To maximize $R(p^o_t, P_c)$, $P_c = 1$ if $p^o_t\alpha(v+1)-\alpha > 0$, and $P_c = 0$ if $p^o_t\alpha(v+1)-\alpha < 0$, that is, eq.(43) as shown at the top of the next page.

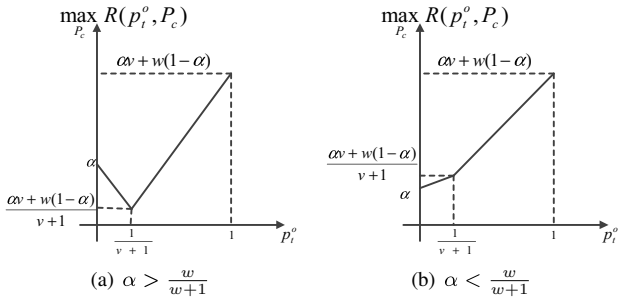

Fig. 2. Illustration of the maximum Bayesian risk. We can find that we can minimize the maximum Bayesian by choosing $p^o_t = \frac{1}{v+1}$ if $\alpha > \frac{w}{w+1}$ and $p^o_t = 0$ if $\alpha < \frac{w}{w+1}$.

Therefore, the maximum Bayesian risk has minimum value $\frac{\alpha v + w(1-\alpha)}{v+1}$ when $p^o_t = \frac{1}{v+1}$. Fig. 2 illustrates the value of eq.(43). We can find that the maximum access probability based on min-max criterion is $\frac{1}{v+1}$. If the value of α is smaller than $\frac{\alpha v + w(1-\alpha)}{v+1}$, then the $p^o_t = 0$. Therefore, $p^o_t = \frac{1}{v+1}$ if $\alpha \geq \frac{w}{w+1}$ else $p^o_t = 0$. ■

B. With Cooperative Sensor

We focus on two specific Bayesian risk functions when we discuss about the scenario that secondary transmitter has cooperative sensor. Let $p_{t|\mathbf{1}(S_C,r_d)=1}$ and $p_{t|\mathbf{1}(S_C,r_d)=0}$ be access probability conditioned on $\mathbf{1}(S_C, r_d) = 1$ and $\mathbf{1}(S_C, r_d) = 0$, respectively. We mainly focus on the *posteriori* probability of existence of primary transmitters after the receiving feedback information. We can replace α in eq.(5) with p and q as follows if $\mathbf{1}(S_C, r_d) = 1$,

$$R(T, P_c|\mathbf{1}(S_C, r_d) = 1) = pvP_c + (1-p)w \tag{44}$$
$$R(NT, P_c|\mathbf{1}(S_C, r_d) = 1) = p(1-P_c), \tag{45}$$

if $\mathbf{1}(S_C, r_d) = 0$,

$$R(T, P_c|\mathbf{1}(S_C, r_d) = 0) = qvP_c + (1-q)w \tag{46}$$
$$R(NT, P_c|\mathbf{1}(S_C, r_d) = 0) = q(1-P_c). \tag{47}$$

We can obtain similar results as that of *Proposition* 3. We omit the detailed proof due to limit of space.

Proposition 4: *The access probability $p^{CH}_{t|\mathbf{1}(S_C,r_d)=1}$ and $p^{CH}_{t|\mathbf{1}(S_C,r_d)=0}$ satisfy*

$$p^{CH}_{t|\mathbf{1}(S_C,r_d)=1} = \begin{cases} \dfrac{1}{v+1}, & \text{if } p \geq \dfrac{w}{w+1} \\ 0, & \text{if } p < \dfrac{w}{w+1}. \end{cases} \tag{48}$$

$$p^{CH}_{t|\mathbf{1}(S_C,r_d)=0} = \begin{cases} \dfrac{1}{v+1}, & \text{if } q \geq \dfrac{w}{w+1} \\ 0, & \text{if } q < \dfrac{w}{w+1}. \end{cases} \tag{49}$$

In Fig. 2, we can find that the maximum risk function is closely related to the *posteri* probability of existence of primary transmitter. That is, if there is no cooperative sensor, it is determined by the value of α. Otherwise, it is determined

$$\max_{P_c} R(p_t^{Het}, P_c) = \begin{cases} p_t^{Het}(\alpha v + w(1-\alpha)), & \text{if } p_t^{Het} \geq \dfrac{1}{v+1} \\ p_t^{Het}(w - \alpha(w+1)) + \alpha, & \text{if } p_t^{Het} < \dfrac{1}{v+1}. \end{cases} \tag{43}$$

by the values of p and q. If $q \geq \frac{w}{w+1}$, it means that secondary transmitters can always transmit data with probability $\frac{1}{v+1}$ (if there is no primary trasnmiter that is close to the secondary transmitters) if $\mathbf{1}(S_C, r_d) = 1$. If $q < \frac{w}{w+1}$ and $p \geq \frac{w}{w+1}$ are satisfied, secondary transmitters can transmit data with probability $\frac{1}{v+1}$ or 0 if feedback information is $\mathbf{1}(S_C, r_d) = 1$ or $\mathbf{1}(S_C, r_d) = 0$. If $p < \frac{w}{w+1}$, then secondary transmitters cannot be successfully decoded by the corresponding secondary receiver.

C. Discussion of Cooperative Sensor

From the definitions in Section III-C, we can provide the following theorem in the heterogeneous scenario.

Theorem 2: *With minimax criterion, cooperative sensor is unnecessary if*

$$\alpha > \frac{w\gamma}{1-\beta+w\gamma},$$

and is necessary if

$$\frac{w(1-\gamma)}{1-\beta+w\gamma} < \alpha < \frac{w\gamma}{\beta+w(1-\gamma)},$$

and is useless if

$$\alpha < \frac{w(1-\gamma)}{\beta+w(1-\gamma)}$$

The access probability is $\frac{1}{v+1}$ if secondary transmitter decides to transmit data.

Proof: According to eq.(48) and (49), we find that S_T will always use the same access probability (cooperative sensor is *unnecessary*) if $q \geq \frac{w}{w+1}$. That is,

$$q = \frac{\alpha(1-\beta)}{\alpha(1-\beta)+(1-\alpha)\gamma} \geq \frac{w}{w+1}. \tag{50}$$

And the cooperative sensor is *useless* if

$$p = \frac{\alpha\beta}{\alpha\beta+(1-\alpha)(1-\gamma)} < \frac{w}{w+1}. \tag{51}$$

The concludes proof. ■

V. The Meaning of Bayesian Risk

In this section, we discuss the physical meaning of the Bayesian risk. Specifically, we show that the values of w and v are related to the maximum probability of collision with primary transmitters and secondary transmitters without S_C, respectively.

We first focus on the meaning of the w. We define the collision probability with primary transmitters P_c^p at the NE as

$$P_c^p \triangleq \mathbb{P}(\mathbf{1}(S_R, r_p) = 0 | \mathbf{1}(S_T, r_d) = 1) p_t^B. \tag{52}$$

We have following result:

Proposition 5: *Without the cooperative sensor, the collision probability with primary users P_c^p is obtained by $P_c^p \leq \frac{1}{w+1}$ if $\alpha \geq \frac{w}{w+1}$.*

Proof: Without the cooperative sensor, $p_t^B \neq 0$ if and only if $\alpha \geq \frac{w}{w+1}$, we get

$$\begin{aligned} P_c^p &= \mathbb{P}(\mathbf{1}(S_R, r_p) = 0 | \mathbf{1}(S_T, r_d) = 1) p_t^B \\ &= (1-\alpha) p_t^B \\ &\leq \lim_{\lambda_s \to 0} (1-\alpha) p_t^B \\ &= 1-\alpha \\ &\leq \frac{1}{w+1}. \end{aligned} \tag{53}$$

The first inequality follows from the fact that the access probability will be much more active when the there is less number of secondary transmitters in the network. The last inequality follows from the fact that $p_t^B \neq 0$ if and only if $\alpha \geq \frac{w}{w+1}$. ■

Therefore, we can modify w to adjust the protection for primary users. For example, if we require $P_c^p \leq \eta_p$, we can set $w = \frac{1-\eta_p}{\eta_p}$. Similarly, we can also modify v to satisfy the constraint on the collision probability with secondary users P_c under the heterogeneous scenario.

Proposition 6: *The collision probability with secondary users at the NE P_c^B is given by $P_c^B \leq \frac{1}{v+1}$ if $\alpha \geq \frac{w}{w+1}$*

Proof: From eq.(11), we know that the $P_c^B = \frac{\alpha-(1-\alpha)w}{\alpha(v+1)}$, and can be further expressed as

$$\begin{aligned} P_c^B &= \frac{\alpha(1+w)-w}{\alpha(v+1)} \\ &= \frac{1+w}{v+1} - \frac{w}{\alpha(v+1)}. \end{aligned} \tag{54}$$

We can find that P_c^B is a monotonic increasing function of α. Therefore, we have

$$P_c^B \leq \max_{\alpha=1} P_c^B = \frac{1}{v+1}. \tag{55}$$

■

Therefore, the value of v can be determined by the probability of collision with other secondary users, that is, $P_c < \eta_s$ and $v = \frac{1-\eta_s}{\eta_s}$.

In the heterogeneous scenario, it is generally difficult for us to guarantee the probability of collision with secondary users because secondary transmitters cannot know the density of secondary transmitters. But we can observe that $\frac{1}{v+1}$ is equivalent to the access probability of secondary users, we can guarantee the access probability of secondary transmitters using eq.(55).

VI. Numerical Result

In this section, we will describe how to apply the results of previous section to decide whether S_T needs to use cooperative

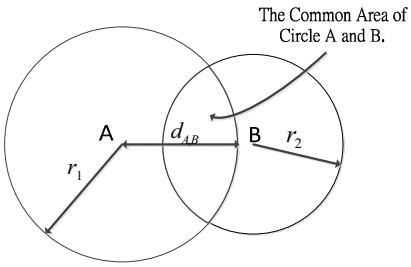

Fig. 3. $\mathcal{B}_c(d_{A,B}, r_1, r_2)$ is the common area of two circles with radius r_1 and r_2 with distance $d_{A,B}$.

sensor or not and how cooperative sensor helps to increase the *transmission allowable* region of S_T.

A. Learning From Experience

Because the information at S_R is not available for S_T, S_T cannot know the value of α. But it is reasonable to assume that S_T can learn the value of α through previous experience. Here, we provide a simple way for secondary transmitters to learn the values of α, β and γ from past experience. By observing N times and count the times of $\mathbf{1}(S_T, r_d) = 1$, S_T can estimate α. Assuming that each observation *i.i.d* and primary transmitters are mobile, the problem of estimating the probability of succession/failures trial can be solved by *Laplace*'s Rule of Succession.

Proposition 7: If all the observation are i.i.d, the estimated a priori probability α is

$$\alpha = \frac{n+1}{N+2}, \tag{56}$$

where n is the number of observation when $\mathbf{1}(S_R, r_p) = 1$. The value of β and γ can be similarly estimated by *Proposition 7*.

We provide the pseudo code of the algorithm in Algorithm 1.

Algorithm 1 Cooperative Sensor Selection Algorithm

for Each possible S_C of S_T **do**
  S_T accesses the channel and estimates $\hat{\alpha}$ by *Proposition 7*;
  S_C feedbacks information $\mathbf{1}(S_C, r_d)$ to S_T;
  S_T estimates $\hat{\beta}$ and $\hat{\gamma}$ by *Proposition 7*;
end for
if S_T in the homogeneous environment **then**
  S_T determines S_C useful or not and access probability according to *Theorem* 1;
end if
if S_T in the heterogeneous environment **then**
  S_T determines S_C useful or not and access probability according to *Theorem* 2;
end if

B. Impact of Primary Users Network

We consider the value of α, β and γ in the random geometric graph model. Because of homogeneous distribution of all the primary users, according to [35], the number of primary users in a region $\mathcal{B}(A, r)$ follows the Poisson distribution with parameters $\lambda_p|\mathcal{B}(A, r)|$. We use the notation $\mathcal{B}_c(d_{A,B}, r_1, r_2)$ to denote the common region of two circles A, B with distance $d_{A,B}$ and radius r_1, r_2 respectively (Fig. 3). The value of α is actually the probability that there is no primary users in the region $\mathcal{B}(S_R, r_p) - \mathcal{B}_c(d_{T,R}, r_d, r_p)$, where $d_{T,R}$ is the distance between S_T and S_R respectively (and we use $d_{T,C}$, $d_{R,C}$ to denote the distance between S_T-S_C and S_R-S_C in the following). In this way, we can express α as follow

$$\begin{aligned}\alpha &= \mathbb{P}(\mathbf{1}(S_R, r_p) = 1 | \mathbf{1}(S_T, r_d) = 1) \\ &= e^{-\lambda_p \pi r_p^2} e^{\lambda_p |\mathcal{B}_c(d_{T,R}, r_d, r_p)|}.\end{aligned}$$

β is the probability of no primary users in the region $\mathcal{B}(S_C, r_d)$ conditioned on no primary users in $\mathcal{B}(S_T, r_d)$ and $\mathcal{B}(S_R, r_p)$. γ is the probability of at least one primary users in the region $\mathcal{B}(S_C, r_d)$ conditioned on primary users in $\mathcal{B}(S_T, r_d)$ but at least one in $\mathcal{B}(S_R, r_p)$. Therefore, we can express β and γ respectively as follow

$$\begin{aligned}\beta &= \mathbb{P}(\mathbf{1}(S_C, r_d) = 1 | \mathbf{1}(S_T, r_d) = 1, \mathbf{1}(S_R, r_p) = 1) \\ &= e^{-\lambda_p |\mathcal{B}(S_C, r_d)|} e^{\lambda_p |\mathcal{B}_c(d_{T,C}, r_d, r_d) \cup \mathcal{B}_c(d_{R,C}, r_p, r_d)|} \\ \gamma &= \mathbb{P}(\mathbf{1}(S_C, r_d) = 0 | \mathbf{1}(S_T, r_d) = 1, \mathbf{1}(S_R, r_p) = 0) \\ &= 1 - \mathbb{P}(\mathbf{1}(S_C, r_d) = 1 | \mathbf{1}(S_T, r_d) = 1, \mathbf{1}(S_R, r_p) = 0) \\ &= 1 - \frac{e^{\lambda_p |\mathcal{B}(S_T, r_d) \cup \mathcal{B}(S_C, r_d)|} - \beta e^{-\lambda_p |\mathcal{B}(S_T, r_d) \cup \mathcal{B}(S_R, r_p)|}}{e^{-\lambda_p |\mathcal{B}(S_T, r_d)|} \left(1 - e^{-\lambda_p (|\mathcal{B}(S_R, r_p) - \mathcal{B}_c(d_{T,R}, r_d, r_p)|)}\right)}.\end{aligned}$$

With the statistical information α, β and γ, S_T can use the *Theorem* 1 and 2 to determine whether to use feedback information from S_C.

In the simulation, we use the parameter setting as $N = 200$, $r_d = 10$, $r_p = 8$, $\lambda_p = 2.5 \times 10^{-3}/m^2$, $\lambda_s = 5\lambda_p$, $w = 9$, $v = \frac{w}{2}$, $S_T = (0, 0)$, $S_C = (2, 0)$.

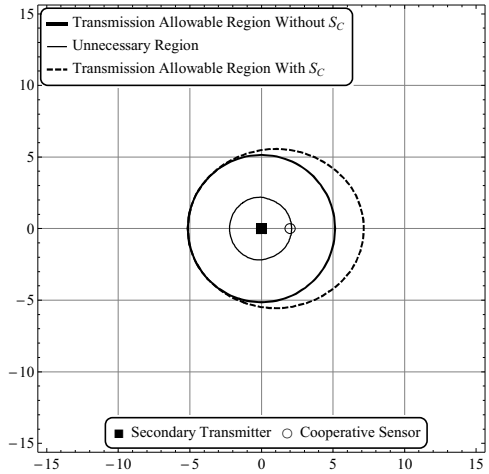

Fig. 4. Homogeneous scenario: All the secondary users use cooperative sensor with the competition among secondary users.

C. Illustration of Transmission Allowable Region

We consider the scenario that all secondary users have complete information about the density of secondary transmitters. From Fig. 4, we can find that the *transmission allowable*

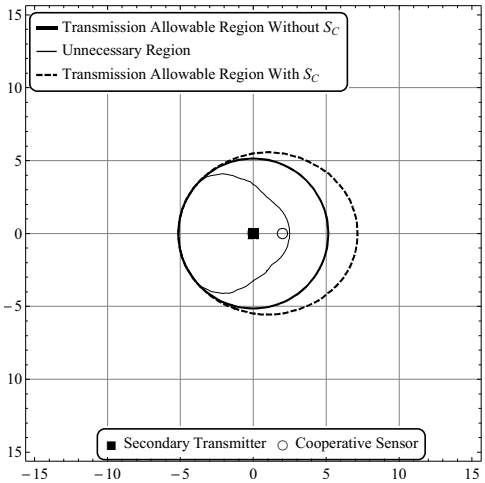

Fig. 5. Homogeneous scenario: There is no competition among secondary transmitters ($\lambda_s \to 0$). The *unnecessary* region becomes larger than that of with competition. But *transmission allowable* region remains the same.

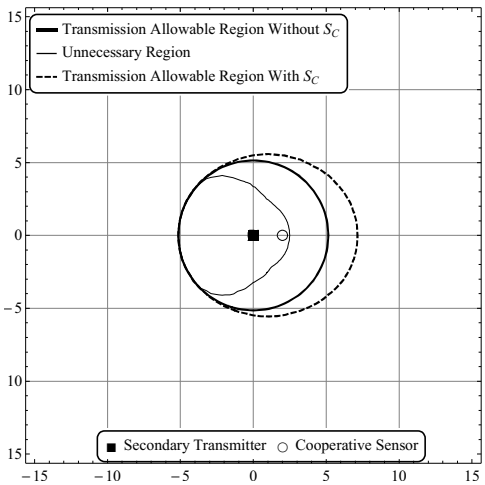

Fig. 6. The transmission region is similar to the scenario that we ignore the effect of other secondary users. The major difference is that secondary transmitters can access spectrum with probability $\frac{1}{v+1}$ in the *transmission allowable* region.

region can be approximated by a circle if all the secondary users have no cooperative sensor due to the homogeneous distribution of primary users. As we consider the existence of the cooperative sensor, we can find that the *transmission allowable* region is generally not symmetric. This is because the communication link is not bi-directional anymore. This is the reason that S_C cannot provide the accurate information about primary users nearby S_R to S_T the cooperative sensor is far from S_R. Therefore, the extended *transmission allowable* region is always located at the side of S_C. If we consider the scenario with $\lambda_s \to 0$, that is, competition-free among secondary users, we get the result in Fig. 5. An interesting observation is that the *transmission allowable* region is the same as that of scenario that there is competition among secondary users. In other words, the *transmission allowable* region of secondary transmitters is mainly determined by the activity of primary users and is independent with that of secondary transmitters. Secondary transmitters can transmit only when there are enough remaining resource that can be shared with primary users. On the other hand, the *unnecessary* region of Fig. 4 is smaller than that of Fig. 5. This verifies our intuition that the cooperative sensor is more important when the competition for transmission opportunity is more intense.

Fig. 6 illustrates the *necessary* and *unnecessary* regions of the secondary transmitter if secondary transmitters have no knowledge about the density of secondary transmitters. We can find that the region is the same as in the competition-free case in homogeneous scenario. The different is that, the access probability is $\frac{1}{v+1}$ if it is not 0. If the secondary receiver is located in the *unnecessary* region, secondary transmitter can always transmit data with probability $\frac{1}{v+1}$ conditioned on it does not detect any nearby primary users around itself. However, if the receiver is located in the *necessary* region, the transmitter accesses the spectrum with probability $\frac{1}{v+1}$ if $\mathbf{1}(S_C, r_d) = 1$, and 0 if $\mathbf{1}(S_C, r_d) = 0$. Due to the unknown of the environment, S_T can use a less aggressive strategy to access channel. On the other hand, if we compare Fig. 4 and Fig. 6, we can find that the information about the density of secondary users does not increase the *transmission allowable* region. This is the reason that the region of CR networks is mainly determined by the primary users.

D. Probability of Successful Transmission Analysis

We define the performance metric as the probability of successful transmission (PST) conditioned on $\mathbf{1}(S_T, r_d) = 1$.

$$PST \triangleq P(\text{successful transmission}|\mathbf{1}(S_T, r_d) = 1). \quad (57)$$

Here we discuss about the PST of the secondary receiver which is located at $(r_s, 0)$. We first express this probability without S_C under the homogeneous scenario as

$$PST = \alpha p_t^B \left(1 - P_c^B\right). \quad (58)$$

The PST with S_C is

$$\begin{aligned} PST = & \\ & \Big(\mathbb{P}(\mathbf{1}(S_C, r_d) = 1) p^{CB}_{t|\mathbf{1}(S_C,r_d)=1} p \\ & + \mathbb{P}(\mathbf{1}(S_C, r_d) = 0) p^{CB}_{t|\mathbf{1}(S_C,r_d)=0} q\Big)\left(1 - P_c^{CB}\right). \end{aligned} \quad (59)$$

For the heterogeneous scenario, we can express PST as follows:

$$PST = \alpha p_t^{Het} e^{-\lambda_s p_t^{Het} \pi r_s^2} \quad (60)$$

The PST with S_C is

$$\begin{aligned} & PST \\ & = \Big(\mathbb{P}(\mathbf{1}(S_C, r_d) = 1) p^{CH}_{t|\mathbf{1}(S_C,r_d)=1} p + \\ & \mathbb{P}(\mathbf{1}(S_C, r_d) = 0) p^{CH}_{t|\mathbf{1}(S_C,r_d)=0} q\Big) e^{-\lambda_a \pi r_s^2}, \end{aligned} \quad (61)$$

where

$$\begin{aligned} & \lambda_a = \lambda_s e^{-\lambda_p \pi r_d^2} \Big(\mathbb{P}(\mathbf{1}(S_C, r_d) = 1) p^{CH}_{t|\mathbf{1}(S_C,r_d)=1} + \\ & \mathbb{P}(\mathbf{1}(S_C, r_d) = 0) p^{CH}_{t|\mathbf{1}(S_C,r_d)=0}\Big) \end{aligned}$$

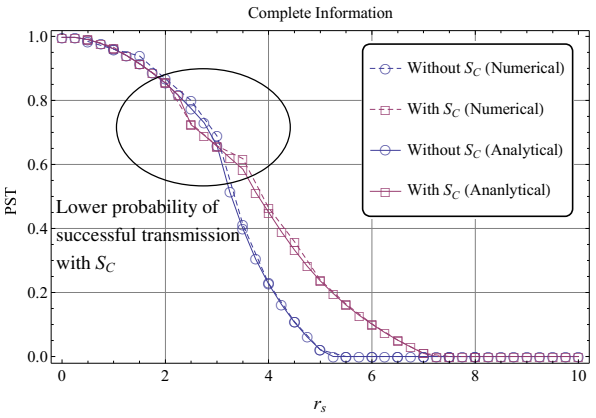

Fig. 7. The illustration of probability of successful transmission under homogeneous scenario. We assume that secondary receiver is located at $(r_s, 0)$. We can find that at r_s about 2 to 3, the PST of with cooperative sensor is lower than that of without the cooperative sensor.

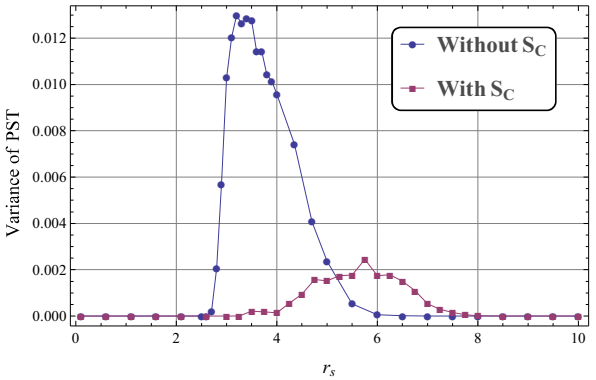

Fig. 8. No matter with cooperative sensor or not, the maximum variance of PST occurs at the edge of transmission allowable region.

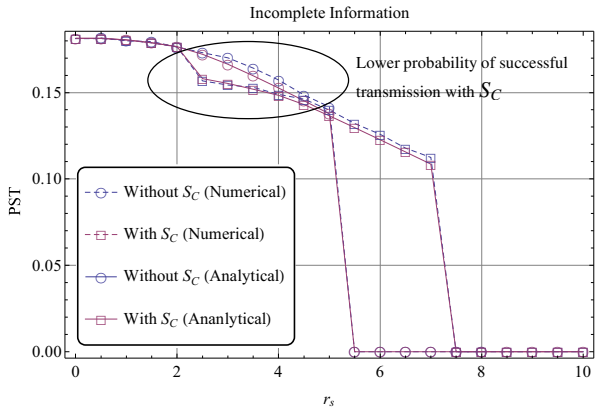

Fig. 9. The illustration of PST under heterogeneous scenario. Without the help of S_C, secondary transmitters can has larger PST in some region.

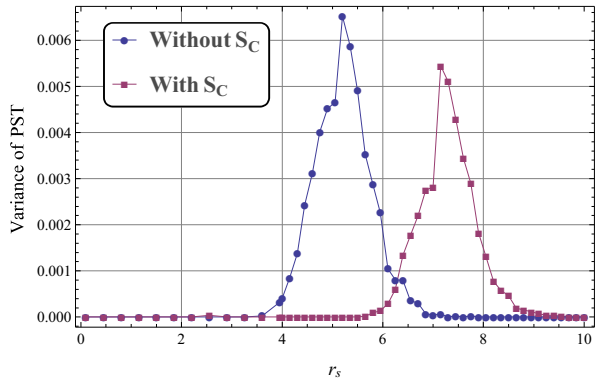

Fig. 10. Similar to homogeneous scenario, the maximum of variance also occurs at the edge of *transmission allowable* region.

Fig. 7 illustrates the PST under the homogeneous scenario. The analytical result reflects the same trends as that of simulation result, which verifies that the approximation of eq.(9) and (24) is accurate. We can find that the cooperative sensor is not always helpful to improve the performance of communication. There is a tradeoff between PST and Bayesian risk (or probability of collision). When the cooperative sensor is *unnecessary* for secondary transmitters, we obtain the same PST no matter with or without the help of cooperative sensor. Cooperative sensor can help secondary transmitters increase the *transmission allowable* region. But we can find that PST of secondary transmitters with cooperative sensor is slightly lower than that of without cooperative sensor in some region. Therefore, there exists tradeoff between collision risk and PST in this region. If the receiver is between the *transmission allowable* region and *unnecessary region*, CR link can have higher PST without the help of S_C. In the homogeneous scenario S_C is helpful for CR communication link only while the receiver is far away from transmitter side.

Fig. 8 illustrates the variance of PST according to different location of receiver. We can find that the maximum variance occurs at the edge of *transmission allowable*. It makes sense because some estimation error may result in access probability to be 0. Similar situation can also be found in heterogeneous scenario.

We present simulation results in the heterogeneous scenario in Fig. 9 and Fig. 10. The maximum variance of PST occurs at the edge of *transmission allowable* region. This is because some deviation of estimation can result in access probability to be $1/(v+1)$ or 0. On the other hand, secondary transmitters without cooperative sensor has better performance than with cooperative sensor when the receiver is located close to the transmitter. Secondary transmitters with cooperative sensor can have better performance only when the receiver is outside of *transmission allowable* region of secondary transmitters without cooperative sensor. Therefore, we can conclude that cooperative sensor cannot always improve the performance of CR networks.

E. Protection for Primary Users

In this subsection, we compare the protection for primary users between the scenario that secondary transmitter takes single primary transmitter and multiple primary transmitters into consideration in a multiple-primary-transmitters environment. If secondary transmitters only take into consideration single primary transmitter, it access the channel if they do not sense any primary transmitters around. This is different from that of our consideration in which secondary transmitters take

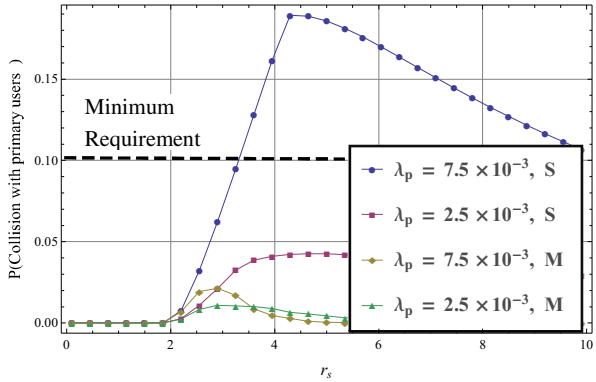

Fig. 11. Homogeneous scenario. We compare the performance of protection for primary users between consideration with single primary transmitter and multiple primary transmitters. S and M refer to consideration with single and multiple primary transmitters respectively. We can find that the proposed approach can guarantee the probability of collision with primary transmitters lower than $w/(w+1)$.

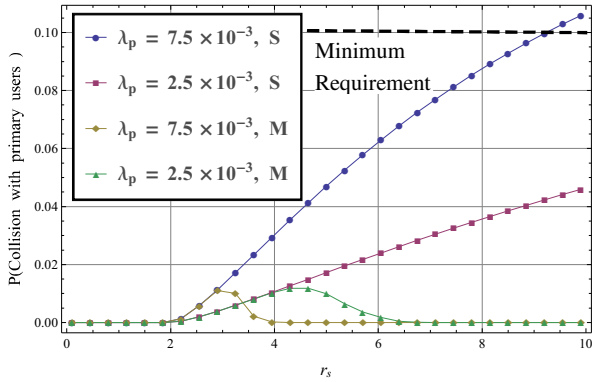

Fig. 12. Heterogeneous scenario. The phenomena similar to that of homogeneous scenario can be found in this scenario. When density of primary transmitters is larger (deviation from single-primary-transmitter environment), the protection for primary users become worse without consideration effect of heterogeneous communication links.

into consideration the heterogeneity of communication links. All the parameters are the same as the previous simulation except density of primary transmitters is 2.5×10^{-3} and 7.5×10^{-3} which refer to sparse and densely environment. The results are shown in Fig. 11 and Fig. 12.

Both of Fig. 11 and Fig. 12 show that the proposed approach can guarantee the probability of collision with primary transmitters to be lower than $w/(w+1)$ either in sparse density or in densely environment. This is the reason that the secondary users achieve the similar performance in the environment with only one primary transmitter and the environment with sparse primary transmitters. However, we cannot guarantee this protection if we do not consider the effect of multiple primary transmitters (that is, heterogeneity of communication link effect), especially in the environment with densely distributed primary transmitters. When the density of primary transmitters is large, the environment deviates from single-primary-user scenario and the heterogeneity of communication links becomes an important issue. Therefore, the proposed approach is promising for providing necessary protection for primary users in environment with multiple primary transmitters.

F. Physical Meaning

With our observation, we have the following guidelines to deploy CRs into an existing (primary) system/network.

1) While we implement CRs into the existing network, the location of the CR receiver is important. If receiver is close to the transmitter (in *unnecessary* region), we do not need to use cooperative sensor (according to *Theorem 1 and 2*). Therefore, for the small cell, such as Pico, Micro, and Femto cell network [39], [40], we may not need to implement additional cooperative sensor.
2) For the network with large coverage area, such as, a cellular network, it is possible that heterogeneous communication links is more perceptible (especially at the edge of service region) than other small cell networks. Most of the receivers may be located out of *unnecessary region*. Then the cooperative sensor may be critical to successful operation of a CR network.
3) The cooperative sensors can increase *transmission allowable region* (spatial reuse efficiency). But we also have to note that cooperative sensors introduce additional communication overhead into the network. The incremental probability of transmission is less (as shown in Fig.7 and 9) if the receiver is nearby the edge of transmission allowable region. Therefore, the tradeoff is important and hence how to achieve the optimal tradeoff in practical system is our future work.
4) On the other hand, the *unnecessary* region can be affected by the density of secondary users. Therefore, how to dynamically control feedback information from cooperative sensor according to the current number of secondary users has the potential to be another solution to reduce the overhead from cooperative sensors.
5) The heterogeneity of communication links is an important issue especially in the environment with multiple primary transmitters. In environment with multiple primary transmitters, an important function of cooperative sensors is to alleviate the sensing error of heterogeneous communication links. This is different from that of cooperative sensors in the environment with single primary transmitter, the function of cooperative sensors focuses on increasing detection accuracy.

VII. Conclusion

To achieve the success of large mobile networks, CR network is important to increase the utilization efficiency of the spectrum. In this work, we study the condition for which cooperative sensor is useful for the secondary users to make the accurate transmission decision from the viewpoint of *transmission allowable region*. We apply the game theoretic model to study the competition among secondary users, we analyze the NE of secondary users. As shown in our analysis and simulation result, we find that the *transmission allowable region* or connection topology of secondary users can be determined by the activities of primary transmitters. On the other hand, the goal of cooperative sensor is to bring

additional information about spectrum resource for transmitter side. However, it can only provide limited benefit to transmitter if transmitter already has enough information to make correct decision.

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