

Carrier Aggregation between Operators in Next Generation Cellular Networks: A Stable Roommate Market

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Abstract—This paper studies carrier aggregation between multiple mobile network operators (MNO), referred to as inter-operator carrier aggregation (IO-CA). In IO-CA, each MNO can transmit on its own licensed spectrum and aggregate the spectrum licensed to other MNOs. We focus on the case that MNOs are distributedly partitioned into small groups, called IO-CA pairs, each of which consists of two MNOs that mutually agree to share their spectrum with each other. We model the IO-CA pairing problem between MNOs as a stable roommate market and derive a condition for which a stable matching structure among all MNOs exists. We propose an algorithm that achieves a stable matching if it exists. Otherwise, the algorithm results in a stable partition. For each IO-CA pair, we derive the optimal transmit power for each spectrum aggregator and establish a Stackelberg game model to analyze the interaction between the licensed subscribers and aggregators in the spectrum of each MNO. We derive the Stackelberg equilibrium of our proposed game and then develop a joint optimization algorithm that achieves the stable matching structure among MNOs as well as the optimal transmit powers for the aggregators and prices for the subscribers of each MNO.

Index Terms—Carrier aggregation, cellular network, cognitive radio, stable roommate, stable marriage, matching, graph, game theory.

I. INTRODUCTION

With the fast growing demand for mobile data service, it becomes more and more difficult to allocate a wide and contiguous frequency band to support high speed data communication for each user equipment (UE). A new technology proposed in LTE-Advanced (LTE-A) [2], referred to as carrier aggregation (CA), allows network operators to support high data rates over large bandwidths by aggregating frequency resources that lie in different frequency bands or which may not be contiguous. The

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next generation of mobile technology will rely on CA to achieve its promised peak data rates. In a typical network, a mobile network operator (MNO) may aggregate frequency resource blocks contiguously or non-contiguously within a single frequency band, i.e. intra-band CA, or it may aggregate resources which are located in separate frequency bands. While much of the current work on CA investigates the aggregation of exclusive blocks of spectrum from the perspective of typical macrocell topologies [3]–[5], we study carrier aggregation (CA) for cellular networks from a cognitive radio (CR) network perspective.

In this paper, we investigate a framework that allows multiple MNOs to access and aggregate each other's licensed spectrum for the purpose of providing more spectrum for the low power elements of their network topology. As such, we propose a system that allows for the dynamic aggregation of spectrum resources over both the MNO's own spectrum holdings and the spectrum holdings of other MNOs. In this scenario, an MNO may operate high power macrocells in its own exclusively licensed spectrum and may dynamically aggregate additional sub-bands, for lower power use, in another network's licensed spectrum. We refer to this type of carrier aggregation as inter-operator CA (IO-CA), i.e. a heterogeneous mix of users, having different rights and employing different transmit powers, exploits the same frequencies over the same area.

From a CR network perspective [6]–[8], each MNO and its corresponding subscriber UEs, who are spectrum license holders, are also referred to as the primary users (PU). The subscriber UEs of each MNO have priority to use their own licensed spectrum, but they can also tolerate a certain interference increase caused by subscriber UEs from other MNOs. These subscriber UEs from other MNOs, also referred to as secondary users (SU), can access the spectrum licensed to the MNO as long as the resulting interference is lower than the tolerable level of the spectrum license holders, i.e. the PUs. We propose two IO-CA approaches: a direct extension of the traditional CA in LTE Advanced into the inter-operator scenario, called regular IO-CA, and a spatial spectrum sharing-based IO-CA framework for multi-operator cellular networks, called sharing IO-CA. In both approaches, each MNO and its subscriber UEs are

regarded as PUs in its own spectrum. Some UEs from each MNO can also access and aggregate the spectrum of other MNOs, where they will be regarded as the SUs and should always control their access to keep the resulting interference under a given tolerable level.

There are several challenges to enable IO-CA between MNOs. Specifically, in the traditional CA within the spectrum of one MNO, the MNO's infrastructure (e.g., eNB in LTE Advanced) controls and manages the spectrum aggregation behavior of its UEs in a centralized fashion. In cellular networks with multiple MNOs, however, there is no central controller and, because of privacy and business reasons, each MNO cannot disclose its private information (such as the payoffs and preference of its UEs and the performance improvement brought by IO-CA) to other MNOs. In addition, since each MNO has already been licensed an exclusive spectrum band and does not have to always rely on IO-CA to achieve the basic quality-of-service (QoS) for its subscriber UEs, each MNO will only allow its spectrum to be aggregated by others when it has an incentive to do so.

One approach that has been proposed in the existing literature is for all the operators to merge their licensed spectrum to form a common spectrum pool [9]–[12]. However, the spectrum pooling system generally requires all operators to give up their exclusive use of spectrum. In addition, the coordination and competition for the spectrum usage among all the operators and their corresponding UEs does not always lead to an efficient solution, especially when the size of the coverage area and the number of operators and UEs becomes large [12]. A tradeoff between spectrum pooling and intra-operator CA can be achieved by partitioning all MNOs into small groups, each of which consisting of a limited number of MNOs that are willing to coordinate and share their spectrum with each other. However, as observed in [8], there may not always exist a stable coalition formation structure in a distributed multi-agent system and even if it exists, there is still a lack of an effective algorithm to allow all MNOs to distributedly negotiate and form this structure. That is, finding a coalition formation structure in a distributed multi-agent system has been proved to be NP-hard [13], [14].

In this paper, we study the joint optimization for an IO-CA-based cellular network with multiple MNOs. We focus on solving four optimization problems:

1) *IO-CA Pairing Problem*: in this problem, we focus on the case that all MNOs can be partitioned into different groups, called IO-CA pairs, each of which consists of two MNOs. In our model, all MNOs can establish a preference over each other and an IO-CA pair can only be established when two MNOs mutually agree to share spectrum with each other. We model this problem as a stable roommate market and seek a stable matching structure among all MNOs such that no MNO or a pair of MNOs has the intention to unilaterally deviate. We allow each MNO to dynamically join or leave an IO-CA pair and

introduce two operations: *deletion* and *addition*, for cellular networks. Specifically, if an MNO observes increasing traffic demands in its network and would like to seek extra network capacity by forming an IO-CA pair with others, it will join the roommate market by using the addition operation to decide its IO-CA pairing partner and still maintain the stability of the existing partitions. Similarly, if the service demand for an MNO in an IO-CA pair decreases to a level that can be satisfied without using IO-CA, the delete operation will be applied to remove this MNO from the existing IO-CA pair. We observe that a stable matching structure may not always exist. We derive a condition for which a stable matching structure exists and propose an algorithm that can detect whether this condition is satisfied and, if it is, achieves this matching structure.

- 2) *Price Adjustment Problem*: in this problem, each MNO charges a price to the subscriber UEs from other MNOs that aggregate its spectrum. We observe that for each MNO, charging a high price to the aggregators will deter the potential MNOs that are willing to form an IO-CA pair. On the other hand, charging a low price will decrease the profit and increase possible interference between pairing MNOs. We hence propose a Stackelberg game-based hierarchical framework to investigate the case where one MNO and its subscriber UEs are the leaders in their own licensed spectrum, with the ability to set prices for secondary use of the spectrum, and subscriber UEs, also called the aggregators, of other MNOs are the followers who can optimize their performance under the prices imposed by the leaders.
- 3) *Power Control Problem*: in this problem, each subscriber UE using sharing IO-CA can optimize its transmit power to further improve its performance without causing intolerably high interference to the subscriber UEs of the primary operator.
- 4) *Joint Optimization problem*: we consider the joint optimization of the above three problems and develop a distributed algorithm to jointly optimize the resulting decisions.

To the best of our knowledge, this is the first work that studies the optimization of IO-CA in cellular networks, adopting a framework that combines the stable roommate market problem and a Stackelberg game.

We observe that not all network systems can support the optimization of all the above problems at the same time. Therefore, in addition to presenting numerical results for our joint optimization algorithm, we also compare the performance improvement brought by each individual solution in our proposed optimization framework.

The remainder of this paper is organized as follows. The background and related works are presented in Section II. The network model and problem formulation are established in Section III. The game theoretic analysis is presented in Section IV. We provide numerical results

and discussions in Section V and conclude the paper in Section VI.

II. BACKGROUND AND RELATED WORKS

Most existing works in inter-operator spectrum sharing focus on cases in which multiple operators share or compete for a common pool of spectrum resources. For example, the authors in [15] have studied power allocation problem for multiple competitive operators coexisting at the same time in the same spectrum. The authors in [16] have applied the two-player non-zero-sum games to study the spectrum allocation problem for multiple operators that share a common spectrum. In [17], the authors have focused on the spectrum allocation problem by assuming all the operators are centrally coordinated and studied a multi-carrier wave-form based inter-operator spectrum sharing concept. In [12], spectrum pooling has been modeled as a hierarchical game and a joint optimization framework has been proposed. Different from these existing works, in our paper, each operator has been licensed an exclusive portion of the spectrum and can autonomously decide whether to share its licensed spectrum with others.

We study the joint optimization problem for cellular networks with multiple MNOs from the game theoretic perspective. Game theory has been widely applied to study distributed optimization problems for spectrum sharing-based CR networks. More specifically, the authors in [18], [19] have introduced a non-cooperative game theoretic model to study the sub-band competition among SUs in a CR network. Stackelberg game-based models have been proposed in [8], [12], [20], [21] to study the interaction between SUs and PUs. In [22], [23], the authors have applied a coalitional game theoretic model to analyze the possible cooperations among different users who share the same spectrum. A detailed survey of game theory and its application into CR networks has been presented in [24]–[26].

We model the pairing problem among MNOs as a stable roommate market. The roommate market and its variations have been extensively studied from both theoretical and practical perspectives [27]–[31]. More specifically, the stable roommate market with ties and incomplete lists has been analyzed in [27]. In [29], the stable roommates market with parallel edges and multiple partners has been considered. A detailed survey for different variants of the stable marriage problem and stable roommate problem has been presented in [30].

III. NETWORK MODEL AND PROBLEM FORMULATION

A. Network Model

Consider a cellular network consisting of a set of closely located MNOs, labeled as $\mathcal{K} = \{M_1, M_2, \dots, M_K\}$ as shown in Figure 1. Each MNO M_i is licensed an exclusive frequency band consisting of a set of component carriers (CCs) each of which can be allocated to support the MNO's UEs. Note that the term "UE" may

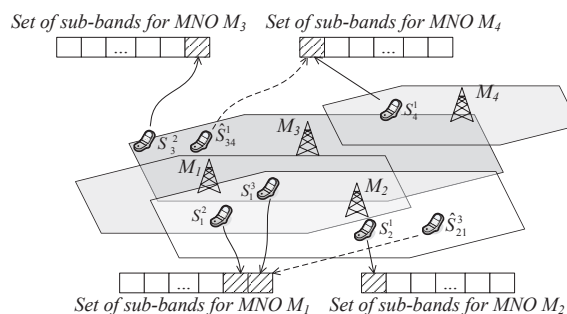


Fig. 1. Network model for an IO-CA-based cellular network with 4 MNOs.

have different meanings in different systems. For example, if each MNO corresponds to a cellular telecommunication network operator, a UE is equivalent to a cellular UE and the corresponding communication channel connecting itself and the infrastructure (e.g., base station). If each MNO corresponds to a device-to-device communication network, each UE then becomes the communication channel between a pair of device-to-device source and destination. Let the set of all UEs currently using the service of MNO M_i be \mathcal{S}_i . It has been recently proposed in [32] that each UE should be able to access more than one CC to support high data rate transmission. In this paper, we use the term "sub-band" to denote the subset of CCs that can be allocated to each UE. Let the set of all sub-bands of MNO M_i be \mathcal{B}_i . The list of notation used in this paper is provided in Table I.

We consider an inter-operator carrier aggregation (IO-CA) system in which a subset of sub-bands of each MNO M_i , denoted as \mathcal{L}_i for $\mathcal{L}_i \subseteq \mathcal{B}_i$, is aggregated by subscriber UEs from other MNOs and, in exchange, a subset $\mathcal{N}_i \subseteq \mathcal{S}_i$ of UEs from MNO M_i can aggregate the sub-bands licensed to other MNOs. We refer to the UEs that subscribe to each MNO as the *subscribers* and those UEs from other MNOs aggregating the spectrum of an MNO as the *aggregators*. Let S_i^k be the subscriber occupying the k th sub-band of MNO M_i . Let \hat{S}_{ij}^l be an aggregator of MNO M_i that accesses the l th sub-band of MNO M_j for $i \neq j$ and $l \in \mathcal{L}_j$. Each MNO needs to first obtain permission from other MNOs before aggregating their spectrum. In most existing network systems, each MNO controls the usage of its sub-bands through its infrastructure (e.g., eNB in LTE systems). Once an MNO M_j agrees to allow other MNOs (e.g., MNO M_i) to aggregate its spectrum, it will allocate each aggregator a specific sub-band, i.e., there is a function mapping each aggregator \hat{S}_{ij}^k to sub-band k of MNO M_j . This function can be centrally decided by the pairing MNOs or it can be the result of sub-band competition between aggregators inside an IO-CA pair [33], [34]. The detailed analysis of the sub-band cooperation or competition among aggregators is outside the scope of this paper. Interested readers can see [8], [18], [19], [33]–[35] for the details.

In this paper, we consider the following two IO-CA approaches:

- 1) *Regular IO-CA*: This approach directly extends the CA in existing LTE-Advanced into the multiple MNOs case where each MNO assigns each aggregator a vacant sub-band that is unoccupied by its subscribers.
- 2) *Sharing IO-CA*: In this approach, each MNO M_i allocates each aggregator to a sub-band that is currently occupied by a subscriber. To maintain the quality of service of the subscriber, M_i should be able to limit the resulting interference caused by each aggregator from other MNOs using a pricing mechanism which will be described in detail in Section IV-B.

In regular IO-CA, the transmission of each aggregator cannot affect the performance of subscribers to the primary MNO, and hence the transmit power of each aggregator \hat{S}_{ij}^k does not need to consider interference constraints. The main shortcoming of regular IO-CA is that the maximum number of aggregators cannot exceed the number of vacant sub-bands. Sharing IO-CA, on the other hand, allows sub-band sharing between aggregators and subscribers and hence can provide further performance improvement even when there are no vacant sub-bands available. The main challenge for sharing IO-CA is that the cross-interference will adversely affect the performance of both sub-band sharing aggregator and subscriber. Therefore, one of the most important issues for sharing IO-CA is interference control [6], [7], [12]. That is, each aggregator \hat{S}_{ij}^k should always maintain its resulting interference to the sub-band sharing subscriber S_j^k below a tolerable level denoted as q_j .

We assume that each IO-CA group can only be formed by two MNOs and refer to a group of MNOs that allow each other to aggregate their sub-bands as an *IO-CA pair*. This assumption is reasonable in practical implementations to make it easy for each MNO to manage interference of the aggregators in its licensed spectrum. More specifically, whenever an MNO detects intolerable interference caused by an aggregator in its sub-band, it will inform the other paired MNO that it should perform interference control or even stop aggregating the sub-band in question. We also assume each sub-band (occupied or unoccupied by a subscriber) can be aggregated by at most one aggregator.

Let the channel gain between source and destination of aggregator \hat{S}_{ij}^k be h_{ij}^k for $i \neq j$ and $k \in \mathcal{L}_j$. Let \mathcal{R}_{ij} be the set of sub-bands of MNO M_j that aggregators from MNO M_i will access using regular IO-CA, with $\mathcal{R}_{ij} \subseteq \mathcal{L}_j$. We consider the following power constraint of each sub-band k for $k \in \mathcal{R}_{ij}$:

$$0 \leq \hat{w}_{ij}^k \leq \tilde{q}_{ij}^k, \quad (1)$$

where \tilde{q}_{ij}^k is the maximum transmit power that can be supported by aggregator \hat{S}_{ij}^k . \hat{w}_{ij}^k is the transmit power of aggregator \hat{S}_{ij}^k in the k th sub-band of MNO M_j . Suppose the rest of the sub-bands in \mathcal{L}_j are aggregated by aggregators from another MNO M_i using sharing IO-CA. We can write the power constraints of sub-band

TABLE I
LIST OF NOTATION

Symbol	Definition
\mathcal{K}	Set of MNOs
M_i	i th MNO
\mathcal{S}_i	Set of UEs for MNO M_i
\mathcal{B}_i	Set of sub-bands licensed to MNO M_i
\mathcal{L}_i	Subset of sub-bands of MNO M_i allowing aggregation from other MNOs
\mathcal{N}_i	Subset of UEs of MNO M_i that can aggregate the spectrum of other MNOs
S_i^k	Subscriber occupying the k th sub-band of MNO M_i
\hat{S}_{ij}^k	Aggregator from MNO M_i aggregating the k th sub-band of MNO M_j
D_i^k	Corresponding destination of S_i^k
B_i^k	Bandwidth of the k th subscriber of M_i
h_{ij}^k	Ratio of the channel gain between aggregator \hat{S}_{ij}^k and subscriber S_j^k to the additive noise received by S_j^k
w_i^k	Transmit power of subscriber S_i^k
\hat{w}_{ij}^k	Transmit power of aggregator \hat{S}_{ij}^k
q_j	Maximum tolerable interference in each sub-band of MNO j
β_i^k	Pricing coefficient charged to the aggregators in the k th sub-band of MNO M_i
$\varpi_{i(j)}$	Part of the payoff of MNO M_i obtained by allowing its spectrum aggregated by aggregators from MNO M_j
$\varpi_{(i)j}$	Part of the payoff of MNO M_i obtained by aggregating the spectrum of MNO M_j
ϖ_{ij}	Total payoff of MNO M_i obtained from an IO-CA pair between M_i and M_j

$k' \in \mathcal{L}_j \setminus \mathcal{R}_{ij}$:

$$0 \leq \hat{w}_{ij}^{k'} \leq \frac{q_j}{h_{ij}^{k'}}, \quad (2)$$

where q_j is the maximum tolerable interference of MNO M_j .

In a regular IO-CA system, each aggregator \hat{S}_{ij}^k can transmit at the maximum power $\hat{w}_{ij}^k = q_i^k$ in its assigned sub-band k and we assume the price paid by \hat{S}_{ij}^k to MNO M_j is a linear function of its transmit power denoted by $\zeta_{(i)j}^k = \beta_j^k \hat{w}_{ij}^k$ ¹. We can hence write the payoff of M_i obtained from regular IO-CA as

$$\varpi_{ij}^R = \sum_{k \in \mathcal{R}_{ij}} \left[B_j^k \log \left(1 + \frac{h_{ii}^k \hat{w}_{ij}^k}{\varrho_i^k} \right) - \zeta_{(i)j}^k \right], \quad (3)$$

where B_i^k is the bandwidth of the k th sub-band of MNO M_i and ϱ_i^k is the additive noise received by \hat{S}_{ij}^k in sub-band k . Note that, in regular IO-CA, each aggregator can only access the vacant sub-band and hence there is no cross interference between the aggregators and subscribers.

In a sharing IO-CA system, the subscribers always have priority to access the sub-bands of their MNOs. More specifically, a subscriber S_i^k in sub-band $k \in \mathcal{L}^i$ can choose its transmit power w_i^k without considering the settings or parameters of the potential aggregators in its

¹The linear pricing function is motivated by the fact that many existing telecommunication mobile systems charge UEs according to their communication data rates, which are monotonically increasing functions of their transmit powers.

sub-band. The price charged by each MNO to aggregators from other MNOs can be in real currency, such as the spectrum rental fee charged by an MNO [36], or it can be in a virtual currency used by each MNO to manage or regulate the interference and accessibility of the aggregators [7], [37].

We assume that each MNO can only control the price of its own sub-bands and should always follow the prices decided by other MNOs when it aggregates their spectrum. In this way, the utility function of each MNO contains two parts:

- 1) The first part consists of the payoff obtained from its own spectrum. We assume each MNO tries to maximize its transmission rates and follow the widely adopted revenue function [8], [12], [35], [38] to define the revenue of MNO M_i obtained from its subscriber S_i^k which shares sub-band $k \in \mathcal{L}_i$ with an aggregator \hat{S}_{ji}^k as

$$\pi_{i(j)}^k(w_i^k, \hat{w}_{ji}^k) = \alpha_i B_i^k \log \left(1 + \frac{h_{ii}^k w_i^k}{\varrho_i^k + h_{ji}^k \hat{w}_{ji}^k} \right). \quad (4)$$

where α_i is the unit price charged by MNO M_i from its subscriber for sending each bit per second of data. Each MNO can also obtain revenue by charging the aggregator in each of its sub-bands. Let the revenue of MNO M_i obtained by charging aggregator \hat{S}_{ji}^k for causing interference to its subscriber S_i^k in sub-band k be $\hat{\pi}_{i(j)}^k(\beta_i^k, \hat{w}_{ji}^k) = \beta_i^k h_{ji}^k \hat{w}_{ji}^k$ where β_i^k is the pricing coefficient of MNO M_i for the aggregator in the k th sub-band for causing each unit of interference on subscriber S_i^k .

We hence can write the first part of the payoff function of MNO M_i in a sharing IO-CA pair formed by M_i and M_j for $M_i \neq M_j$ as follows:

$$\begin{aligned} \varpi_{i(j)}'(\beta_i, \mathbf{w}_i, \hat{\mathbf{w}}_{ji}) &= \sum_{k \in \mathcal{L}_i \setminus \mathcal{R}_{ij}} \varpi_{i(j)}^k(w_i^k, \hat{w}_{ji}^k, \beta_i^k), \end{aligned} \quad (5)$$

where $\varpi_{i(j)}^k(w_i^k, \hat{w}_{ji}^k, \beta_i^k) = \pi_{i(j)}^k(w_i^k, \hat{w}_{ji}^k) + \hat{\pi}_{i(j)}^k(\beta_i^k, \hat{w}_{ji}^k)$, $\beta_i = \{\beta_i^k\}_{k \in \mathcal{L}_i}$, $\mathbf{w}_i = \{w_i^k\}_{k \in \mathcal{L}_i}$ and $\hat{\mathbf{w}}_{ji} = \{\hat{w}_{ji}^k\}_{k \in \mathcal{L}_i}$. Note that if there is no aggregator sharing sub-band k which is currently occupied by subscriber S_i^k , we can write the payoff of MNO M_i obtained from its subscriber in the k th sub-band as $\varpi_{i(j)}^k(w_i^k, \hat{w}_{ji}^k = 0, \beta_i^k)$.

- 2) The second part of the payoff function of MNO M_i consists of the payoff obtained by aggregating sub-bands of others. We can write the revenue of MNO M_i obtained from aggregator \hat{S}_{ij}^m using the m th sub-band of M_j as

$$\begin{aligned} \hat{\pi}_{(i)j}^m(w_j^m, \hat{w}_{ij}^m) &= \alpha_i' B_j^m \log \left(1 + \frac{h_{ii}^m \hat{w}_{ij}^m}{\varrho_j^m + h_{ji}^m w_j^m} \right), \end{aligned} \quad (6)$$

where α_i' is the price charged by M_i to its subscriber for aggregating sub-band m of MNO M_j .

Each MNO M_i also needs to pay a price when its aggregators access the sub-bands of other MNOs. We hence define the cost of each aggregator $\hat{S}_{ij}^m \in \mathcal{N}_i$ of MNO M_i accessing the m th sub-band of MNO M_j as $\zeta_{(i)j}^m(\beta_j^m, \hat{w}_{ij}^m) = \beta_j^m h_{ij}^m \hat{w}_{ij}^m$.

The second part of the payoff function of MNO M_i in a sharing IO-CA pair can then be written as

$$\begin{aligned} \varpi_{(i)j}'(\beta_j, \mathbf{w}_j, \hat{\mathbf{w}}_{ij}) &= \sum_{S_i^m \in \mathcal{N}_i, m \in \mathcal{L}_j \setminus \mathcal{R}_{ij}} \varpi_{(i)j}^m(\beta_j^m, w_j^m, \hat{w}_{ij}^m), \end{aligned} \quad (7)$$

where $\varpi_{(i)j}^m(\beta_j^m, w_j^m, \hat{w}_{ij}^m) = \hat{\pi}_{(i)j}^m(w_j^m, \hat{w}_{ij}^m) - \zeta_{(i)j}^m(\beta_j^m, \hat{w}_{ij}^m)$.

Note that if there is no subscriber currently occupying sub-band k , i.e., $w_i^k = 0$ and $h_{ji}^k = 1$, the resulting payoff $\varpi_{(i)j}^m(\beta_j^m, w_j^m, \hat{w}_{ij}^m)$ will become equivalent to the payoff of regular IO-CA in (3), i.e., we can also write $\varpi_{ij}^R = \sum_{k \in \mathcal{R}_{ij}} \varpi_{(i)j}^m(\beta_j^m, w_j^m = 0, \hat{w}_{ij}^m)$.

By combining (5) and (7), we can write the total payoff of MNO M_i when it forms an IO-CA pair with MNO M_j as

$$\begin{aligned} \varpi_{ij}(\beta_i, \beta_j, \mathbf{w}_i, \hat{\mathbf{w}}_{ji}, \mathbf{w}_j, \hat{\mathbf{w}}_{ij}) &= \varpi_{i(j)}(\beta_i, \mathbf{w}_i, \hat{\mathbf{w}}_{ji}) + \varpi_{(i)j}(\beta_j, \mathbf{w}_j, \hat{\mathbf{w}}_{ij}). \end{aligned} \quad (8)$$

where

$$\begin{aligned} \varpi_{i(j)}(\beta_i, \mathbf{w}_i, \hat{\mathbf{w}}_{ji}) &= \sum_{k \in \mathcal{R}_{ij}} \varpi_{i(j)}^k(w_i^k, \hat{w}_{ji}^k = 0, \beta_i^k) + \sum_{k \in \mathcal{L}_{ij} \setminus \mathcal{R}_{ij}} \varpi_{i(j)}^k(w_i^k, \hat{w}_{ji}^k \neq 0, \beta_i^k) \quad \text{and} \\ \varpi_{(i)j}(\beta_j, \mathbf{w}_j, \hat{\mathbf{w}}_{ij}) &= \sum_{S_i^m \in \mathcal{N}_i, m \in \mathcal{R}_{ij}} \varpi_{(i)j}^m(\beta_j^m, w_j^m = 0, h_{ij}^m = 1, \hat{w}_{ij}^m) + \sum_{S_i^m \in \mathcal{N}_i, m \in \mathcal{L}_j \setminus \mathcal{R}_{ij}} \varpi_{(i)j}^m(\beta_j^m, w_j^m, \hat{w}_{ij}^m). \end{aligned}$$

Note that if an MNO M_i decides to use neither regular nor sharing IO-CA, the resulting payoff is only related to the transmit power of each subscriber, i.e., we have

$$\varpi_{ii}(\varpi_i) = \varpi_{ij}(\mathbf{w}_i, \hat{\mathbf{w}}_{ji} = 0, \mathbf{w}_j = 0, \hat{\mathbf{w}}_{ij} = 0). \quad (9)$$

It can be observed that sharing IO-CA is particularly useful in heterogeneous networks with multi-tiers in which the network of each MNO consists of both macro-cells with high-power infrastructure and low powered operator- or user-deployed small-cells or femto-cells. In this case, allowing the high-power infrastructure of an MNO to share the same spectrum as the small-cell infrastructure of the same MNO will have the potential to cause large interference to both the macro-cell and small-cell users. On the other hand, allowing the low power infrastructure of one MNO to aggregate the spectrum used by the high power infrastructures of another MNO can alleviate the cross-tier interference. We will provide a more detailed discussion of this scenario using simulation results in Section V.

B. Problem Formulation

In cellular network systems, different MNOs have different infrastructure and spectrum resources and always

have the incentive to maximize their performance by taking full advantage of the aggregated spectrum of other MNOs. This makes it natural to study the IO-CA from the game theoretic perspective. In this paper, we assume each MNO is selfish and can strategically decide its parameters to optimize its performance and will only seek cooperation with other MNOs when this cooperation can provide mutual benefits. In this paper, we focus on the following problems,

- 1) *Pairing Problem*: In an IO-CA-based cellular network, each MNO is selfish and can always make autonomous decisions about its IO-CA partner. More specifically, each MNO M_i needs to send a request to another MNO (e.g., M_j) and an IO-CA pair between M_i and M_j can only be established when the request is accepted by M_j . To attract other MNOs to aggregate its spectrum, each MNO should also reveal some information such as the set of sub-bands allowed for aggregation and the prices charged to each aggregator, for other MNOs to evaluate the achievable performance in an IO-CA pair. If MNO M_j in an IO-CA pair can further improve its payoff by pairing with a different MNO (e.g., M_k for $k \neq i, j$) which will also accept the request from M_i , the IO-CA pair between M_i and M_j will not be stable. Therefore, it is important to decide a stable IO-CA pairing structure in which every MNO sticks to its IO-CA pairing partner and has no intention to unilaterally deviate. Note that the pairing request signals sent by each MNO only need to include the identity information of each MNO and therefore, the communication overhead caused by sending and responding to the pairing request between two pairing MNOs can be regarded as a small constant which is neglected in this paper.
- 2) *Pricing Adjustment Problem for Each MNO in Its Licensed Spectrum*: Each MNO can control the price charged to the spectrum aggregators in its licensed spectrum. Therefore, the pricing adjustment problem for each MNO M_i in an IO-CA pair formed between M_i and M_j can be written as

$$\max_{\beta_i} \varpi_{ij}(\beta_i, \beta_j^*, \mathbf{w}_i, \hat{\mathbf{w}}_{ji}, \mathbf{w}_j, \hat{\mathbf{w}}_{ij}). \quad (10)$$

where β_j^* is the optimal price of MNO M_j .

- 3) *Power Control Problem for Each MNO in the Spectrum of Others*: When accessing the spectrum of other MNOs, each MNO can use power control methods to further improve its performance given the prices imposed by the spectrum license holders. The power optimization problem for each aggregator of an MNO M_i can be formulated as

$$\max_{\hat{\mathbf{w}}_{ij}} \varpi_{ij}(\beta_i, \beta_j, \mathbf{w}_i, \hat{\mathbf{w}}_{ji}^*, \mathbf{w}_j, \hat{\mathbf{w}}_{ij}). \quad (11)$$

where $\hat{\mathbf{w}}_{ji}^*$ are the optimal transmit powers of the aggregators from MNO M_j .

- 4) *Joint Optimization Problem*: In this paper, we consider the joint optimization of the above three

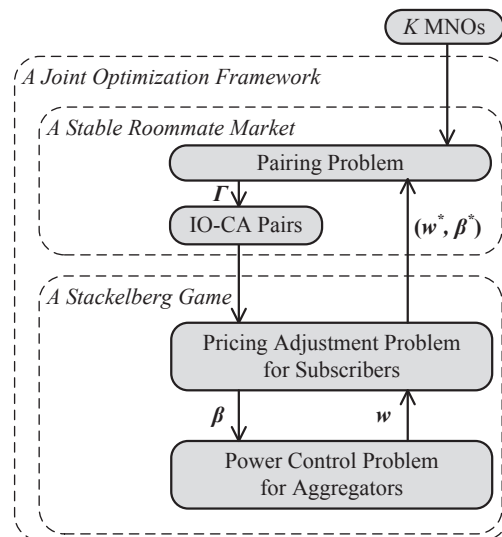


Fig. 2. Relationship among different problems and game models in an IO-CA-base cellular network.

problems. More specifically, we model the pairing problem as a stable roommate market and seek a stable matching structure among all the MNOs. We then establish a Stackelberg game-based hierarchical framework [8] within each IO-CA pair in which, in the licensed spectrum of each MNO M_i , M_i and its corresponding subscribers are the leaders (or primary user, seller, etc.) and the aggregators from the other pairing MNO M_j for $j \neq i$ are the followers (also called secondary user, buyer, etc.). Using this framework, we propose an algorithm to jointly optimize the transmit power, prices, and pairing partner of each MNO and its corresponding subscribers and aggregators.

IV. A JOINT OPTIMIZATION FRAMEWORK FOR AN IO-CA SYSTEM

In this section, we discuss the solutions of the problems described at the end of Section III. We first study the power control problem for an IO-CA pair in Section IV-A. We then discuss Stackelberg game modeling and the joint price and transmit power optimization problem in Section IV-B. Finally, we establish a stable roommate market to study the IO-CA pairing problem for a network with three or more MNOs in Section IV-C. The relationship between different models in our joint optimization framework is illustrated in Figure 2.

A. Optimal Power Control for each MNO

Once an IO-CA pair has been established and a sub-band has been aggregated by an aggregator, it is important for each aggregator to maintain the resulting interference below the tolerable levels defined in (1) and (2). In this subsection, we assume an IO-CA pair has already been formed between MNOs M_i and M_j for $i \neq j$ and $M_i, M_j \in \mathcal{K}$ and pricing coefficients β_i and β_j for both MNOs are constants. Let

us consider the power control of an aggregator \hat{S}_{ij}^k from MNO M_i aggregating the k th sub-band of MNO M_j . The optimization of the pricing coefficients and pairing partners for MNOs will be discussed in the next two subsections. Following the same line as Section III, we can rewrite the optimization of the power control problem for MNO M_i as follows:

$$\begin{aligned} \max_{\hat{w}_{ij}} \quad & \varpi_{ij}(\beta_i, \beta_j, \mathbf{w}_i, \hat{\mathbf{w}}_{ji}^*, \mathbf{w}_j, \hat{w}_{ij}) \\ \text{s.t.} \quad & h_{ij}^k \hat{w}_{ij}^k \leq q_j, h_{ji}^l \hat{w}_{ji}^{l*} \leq q_i, \hat{w}_{ij}^k \leq \tilde{q}_j^k \\ & \text{and } \hat{w}_{ji}^l \leq \tilde{q}_i^l \quad \forall k \in \mathcal{L}_j, l \in \mathcal{L}_i \\ & w_i^l, w_j^k, \hat{w}_{ij}^k, \hat{w}_{ji}^{l*} \geq 0, \\ & \forall k \in \mathcal{L}_j, l \in \mathcal{L}_i, S_i^l \in \mathcal{N}_i, S_j^k \in \mathcal{N}_j. \end{aligned} \quad (12)$$

As the transmissions in different sub-bands are independent with each other and the payoff function $\varpi_{(ij)}^k(w_j^k, \hat{w}_{ij}^k, \beta_j^k)$ of aggregator \hat{S}_{ij}^k from MNO M_i is concave in \hat{w}_{ij}^k for a given β_j^k and w_j^k , we can then derive the following optimal transmit power \hat{w}_{ij}^{k*} of each aggregator \hat{S}_{ij}^k of MNO M_i by setting $\frac{\partial \varpi_{(ij)}^k(w_j^k, \hat{w}_{ij}^k)}{\partial \hat{w}_{ij}^k} = 0$ where \hat{w}_{ij}^{k*} is given by,

$$\hat{w}_{ij}^{k*} = \left(\min \left\{ \left(\frac{B_j^k}{\beta_j^k h_{ij}^k} - \frac{q_i^k + h_{ji}^k w_j^k}{h_{ii}^k} \right), \tilde{q}_j^k, \frac{q_j}{h_{ij}^k} \right\} \right)^+, \quad (13)$$

where $(\cdot)^+ = \max\{0, \cdot\}$. We can write the optimal transmit power of all aggregators of MNO M_i as $\hat{\mathbf{w}}_{ij}^* = \{\hat{w}_{ij}^{k*}\}_{k \in \mathcal{L}_j}$ where \hat{w}_{ij}^{k*} is given in (13).

It can be observed that aggregator \hat{S}_{ij}^k can only calculate the optimal transmit power by knowing the pricing coefficient β_j^k , the channel gain and transmit power w_j^k . In a practical system, each MNO M_i will rely on the other paired MNO M_j to provide this information. If M_j refuses to disclose such information to M_i , \hat{S}_{ij}^k cannot determine the optimal transmit power but has to send signals using a pre-defined fixed power. In Section V, we compare the performance of an IO-CA-based cellular network with and without using optimal power control.

Another observation from (13) is that the optimal transmit power \hat{w}_{ij}^{k*} of aggregator \hat{S}_{ij}^k decreases with the pricing coefficient β_j^k . In other words, each MNO M_j can control the interference level of the aggregator \hat{S}_{ij}^k in each of its sub-bands by adjusting the corresponding pricing coefficient β_j^k . We will provide a more detailed discussion on how each MNO decides the optimal pricing coefficient in the next subsection.

We can prove the following result:

Proposition 1: If $\exists k \in \mathcal{L}_j$, $\hat{w}_{ij}^{k*} > 0$, then $\varpi_{(ij)} > 0$.

Proof: See Appendix A. ■

The above proposition says that if the optimal transmit power of at least one of aggregators from MNO M_i is positive, MNO M_i can always obtain benefits by aggregating the k th sub-band of MNO M_j . Note that if

$\hat{w}_{ij}^{k*} = 0$, it means that \hat{S}_{ij}^k cannot aggregate the k th sub-band of MNO M_j .

B. Optimal Price and Stackelberg Game for each MNO

Let us consider the joint optimization of the transmit powers and pricing coefficients for an IO-CA pair formed between MNOs M_i and M_j for $M_i \neq M_j$ and $M_i, M_j \in \mathcal{K}$. By substituting the optimal transmit powers $\hat{\mathbf{w}}_{ij}^*$ and $\hat{\mathbf{w}}_{ji}^*$ in (13) into $\varpi_{ij}(\mathbf{w}_i, \mathbf{w}_j, \beta_i, \beta_j, \hat{\mathbf{w}}_{ij}^*, \hat{\mathbf{w}}_{ji}^*)$, we can observe that the payoff of each MNO depends on the pricing coefficients of both paired MNOs. This means that the pricing optimization problems for both MNOs are correlated. Specifically, the optimal β_i^* decided by M_i affects the optimal transmit powers of the aggregators from MNO M_j , which also determines the pricing coefficients and the incentive of M_j to form an IO-CA pair with MNO M_i . However, each MNO can only control the price of its own spectrum. Recall from Section III, the payoff of each MNO M_i consists of two parts: $\varpi_{i(j)}(\beta_i, \mathbf{w}_i, \hat{\mathbf{w}}_{ji}^*)$ and $\varpi_{(i)j}(\beta_j, \mathbf{w}_j, \hat{\mathbf{w}}_{ij}^*)$. The pricing coefficient β_i decided by M_i can only affect the first part $\varpi_{i(j)}(\beta_i, \mathbf{w}_i, \hat{\mathbf{w}}_{ji}^*)$, and the second part $\varpi_{(i)j}(\beta_j, \mathbf{w}_j, \hat{\mathbf{w}}_{ij}^*)$ depends on the pricing coefficient β_j controlled by M_j .

As each MNO has the autonomy to decide and manage the spectrum usage in its own sub-bands, it can determine the price charged to each aggregator in each of its sub-bands considering that all aggregators will use the optimal transmit powers discussed in Section IV-A. The interactions between users that must decide what actions to take in a sequential manner make it natural to model the above pricing and transmit power optimization problem as a Stackelberg game as follows: in its own licensed spectrum, each MNO is a *leader* and its *action* is to select the pricing coefficient when the strategic aggregators access its spectrum. Each aggregator is a *follower* and its *action* is to optimize the transmit power according to the prices imposed by the MNOs. We seek a Stackelberg equilibrium solution for our proposed game which is formally defined as follows.

Definition 1: [39, Definition 3.26-3.28] Suppose MNOs M_i and M_j form an IO-CA pair. In the spectrum of MNO M_i , M_i is the leader and aggregators from MNO M_j are the followers. An action pair $(\beta_i^*, \hat{\mathbf{w}}_{ji}^*)$ is a Stackelberg equilibrium if $\hat{\mathbf{w}}_{ji}^*$ satisfies

$$\begin{aligned} \varpi_{ij}(\beta_i^*, \beta_j^*, \hat{\mathbf{w}}_{ji}^*, \hat{\mathbf{w}}_{ij}^*, \mathbf{w}_i, \mathbf{w}_j) \\ \geq \varpi_{ij}(\beta_i^*, \beta_j^*, \hat{\mathbf{w}}_{ji}^*, \hat{\mathbf{w}}_{ij}^*, \mathbf{w}_i, \mathbf{w}_j), \quad \forall \hat{\mathbf{w}}_{ji} \in \mathbb{R}^{|\mathcal{L}_i|}, \end{aligned} \quad (14)$$

where $\beta_i^* = \arg \max_{\beta_i \in \mathbb{R}^{|\mathcal{L}_i|}} \varpi_{ij}(\beta_i, \beta_j^*, \hat{\mathbf{w}}_{ji}^*, \hat{\mathbf{w}}_{ij}^*, \mathbf{w}_i, \mathbf{w}_j)$ satisfies

We can prove the following results about the Stackelberg equilibrium for our proposed game.

Theorem 1: For each IO-CA pair formed by MNOs M_i and M_j , $(\beta_i^*, \hat{\mathbf{w}}_{ji}^*)$ is a Stackelberg equilibrium in the licensed spectrum of M_i where $\hat{\mathbf{w}}_{ji}^*$ is given in (13) and $\beta_i^* = \{\beta_i^{k*}\}_{k \in \mathcal{L}_i}$ for β_i^{k*} is given as follows: If

$(2 - 2\rho_i^k + \theta_i^k)^2 < \theta_i^k (\theta_i^k - \rho_i^k)$, $\beta_i^{k*} = \beta_i^{k-}$ where β_i^{k-} is given by

$$\beta_i^{k-} = \frac{h_{ii}^k B_i^k}{h_{ii}^k q_i + h_{ii}^k + h_{ji}^k w_{ji}^{k*}}. \quad (15)$$

and $\rho_i^k = \frac{h_{ji}^k (1 + h_{ij}^k w_{ij}^k)}{h_{jj}^k}$, $\theta_i^k = h_{ii}^k w_i^k$. If $(2 - 2\rho_i^k + \theta_i^k)^2 \geq \theta_i^k (\theta_i^k - \rho_i^k)$, then

$$\beta_i^{k*} = \arg \max_{\substack{\beta_i^{k*} \in \{\beta_i^{k1*}, \beta_i^{k2*}, \beta_i^{k-}, \beta_i^{k+}\} \\ \beta_i^{k*} \in [\beta_i^{k-}, \beta_i^{k+}]}} \{ \varpi_i^k (\beta_i^{k*}, \hat{w}_{ji}^{k*}) \}, \quad (16)$$

where $\beta_i^{k+} = \frac{B_i^k}{\rho_i^k}$ and β_i^{k1*} is given by

$$\beta_i^{k1*} = \frac{B_i^k \sqrt{\rho_i^k \theta_i^k \left((2 - 2\rho_i^k + \theta_i^k)^2 + \theta_i^k (\rho_i^k - \theta_i^k) \right)}}{2\rho_i^k (\rho_i^k - 1) (\rho_i^k - 1 - \theta_i^k)} \quad (17)$$

$$\frac{B_i^k \rho_i^k (2 - 2\rho_i^k + \theta_i^k)}{2\rho_i^k (\rho_i^k - 1) (\rho_i^k - 1 - \theta_i^k)},$$

$$\beta_i^{k2*} = \frac{B_i^k \rho_i^k (2 - 2\rho_i^k + \theta_i^k)}{2\rho_i^k (1 - \rho_i^k) (\rho_i^k - 1 - \theta_i^k)} \quad (18)$$

$$\frac{B_i^k \sqrt{\rho_i^k \theta_i^k \left((2 - 2\rho_i^k + \theta_i^k)^2 + \theta_i^k (\rho_i^k - \theta_i^k) \right)}}{2\rho_i^k (\rho_i^k - 1) (\rho_i^k - 1 - \theta_i^k)}.$$

Proof: See Appendix B. ■

Similarly, we can also observe that $(\beta_j^*, \hat{w}_{ij}^*)$ is the Stackelberg equilibrium solution in the spectrum licensed to MNO M_j where β_j^* and \hat{w}_{ij}^* can be obtained by swapping i and j in Theorem 1 and equation (13).

Note that the optimal pricing coefficients Theorem 1 are calculated by assuming all aggregators use the optimal power control methods derived in (13). If the aggregators use constant power to send signals, both MNOs should charge the highest prices they can in each of their sub-bands to maximize their revenue, i.e., $\beta_i^k \rightarrow \beta_i^{k+} \forall k \in \mathcal{L}_i$.

The optimal pricing coefficient pair (β_i^*, β_j^*) for both paired MNOs M_i and M_j derived in the above theorem also corresponds to the Bertrand equilibrium solution if we model the price competition between the pairing MNOs as an oligopoly market where two market dominant firms compete with each other with different prices [38], [40], [41].

C. A Stable Roommate Market for the IO-CA Pairing Problem

Let us consider the pairing problem for a cellular network with three or more MNOs. In our model, an IO-CA pair can only be formed when two MNOs mutually agree to share their spectrum with each other. This makes it natural to model the interaction among MNOs as a roommate market, also known as one-sided matching market [42] or non-bipartite matching market

[43], in which K students (or, in our model, MNOs) will try to be assigned into $\lceil \frac{K}{2} \rceil$ rooms (or, in our model, IO-CA pairs) each of which accommodates two students. Let us first define the roommate market as follows.

Definition 2: [44, Chapter 4.1] A *roommate market* is specified by a set \mathcal{K} of K students and a preference list P_k for each student k for $k \in \mathcal{K}$. A *preference relation* for the roommate market is a tuple $R = \langle \mathcal{K}, \mathbf{P} \rangle$ where \mathbf{P} is the preference table of all students defined as $\mathbf{P} = \langle P_k \rangle_{k \in \mathcal{K}}$.

We define the IO-CA pairing problem for a cellular network as a stable roommate market, referred to as the *IO-CA market*, in which the students are modeled as MNOs and each MNO M_n has a preference over all the other MNOs that can improve its payoff by forming a IO-CA pair, i.e., we use $P_n(M_i)$ to denote the rank of MNO M_i in the preference list of M_n and $P_n(M_i) < P_n(M_j)$ means $\varpi_{ni}(\mathbf{w}_n, \mathbf{w}_i, \beta_n^*, \beta_i^*, \hat{\mathbf{w}}_{ni}^*, \hat{\mathbf{w}}_{in}^*) > \varpi_{nj}(\mathbf{w}_n, \mathbf{w}_j, \beta_n^*, \beta_j^*, \hat{\mathbf{w}}_{nj}^*, \hat{\mathbf{w}}_{jn}^*)$ for $M_i, M_n, M_j \in \mathcal{K}$. Note that IO-CA cannot always improve the payoff for both paired MNOs and, if an MNO M_i cannot improve its payoff by forming IO-CA with any MNOs in the market, it will not share its spectrum with others but only use its own exclusive spectrum to support services for its subscribers, i.e., if M_n occupies the l th position in the preference list of itself, it means that $\varpi_{ni}(\mathbf{w}_n, \mathbf{w}_i, \beta_n^*, \beta_i^*, \hat{\mathbf{w}}_{ni}^*, \hat{\mathbf{w}}_{in}^*) > \varpi_{nn}(\mathbf{w}_n)$ for all $M_i \in \mathcal{K}$ satisfying $0 < P_n(M_i) < l$.

Different MNOs generally have different peak hours. We hence can assume, in a cellular network, MNOs sequentially join or leave the IO-CA market². If an MNO M_i decides to join the market, it will send a message to inform all the MNOs in the current IO-CA market that the spectrum of M_i will be available to share. MNOs in the market can use the message sent by M_i to evaluate the performance of M_i and insert M_i into the proper positions in their preference lists. All MNOs will also feedback a confirmation message to M_i which can be used by M_i to establish the preference list over all the MNOs. Let \mathcal{P}_i be the preference of M_i . Let \tilde{M}_i^k be the k th most preferred MNO in the preference list of MNO M_i .

One of the main solution concepts for the roommate market is the *matching* which is defined as follows.

Definition 3: [44, Chapter 4.1] A (one-sided) *matching* Γ for a roommate market is a function from sets \mathcal{K} to \mathcal{K} such that $\Gamma(M_i) \in \mathcal{K}$, $\Gamma(M_j) \in \mathcal{K}$, and $\Gamma(M_i) = M_j \Leftrightarrow \Gamma(M_j) = M_i$ for every $M_i, M_j \in \mathcal{K}$.

Note that $\Gamma(M_i) = M_i$ means that M_i cannot form an IO-CA pair with any of the other MNOs. It can be observed that there are $\prod_{i=1}^{K/2} \binom{K-2i}{2} / \frac{K}{2}!$ number of possible matchings where $\binom{m}{n}$ is the number of m combinations from a set of

²If multiple MNOs decide to join the market simultaneously, a random duration of delay can be introduced for these MNOs. That is, if an MNO M_i decides to join the IO-CA market, it will delay for η_i amount of time before sending the joining request where η_i is a bounded random variable. We have included this random delay in Algorithm 1.

of n elements. In this paper, we seek a matching structure that is *stable* which is defined as follows.

Definition 4: A *stable* matching is a partition of set \mathcal{K} into $\lceil \frac{K}{2} \rceil$ disjoint IO-CA pairs such that no two MNOs who are not in the same IO-CA pair but each of whom prefers the other to its partner in the matching.

It has already been observed in [44] that a stable matching for the stable roommate market may not always exist. This is because the preference of each MNO over each other may form a cyclic sequence. For example, it can be easily shown that if the preferences of four MNOs M_1, M_2, M_3 and M_4 are given by $P_1 = \langle M_2, M_3, M_4 \rangle$, $P_2 = \langle M_3, M_1, M_4 \rangle$, $P_3 = \langle M_1, M_2, M_4 \rangle$, and $P_4 = \langle M_1, M_2, M_3 \rangle$, respectively, it is impossible to find a stable matching, e.g., any MNO, for instance M_i , that is matched with M_4 will be able to find another more preferable MNO which also prefers M_i to its current matching MNO.

Another concept called *stable partition*, which can be regarded as a generalization of the stable matching, was first proposed in [45], [46]. It has already been proved in [45] that a stable partition always exists in any instance of the roommate market. Let us present the formal definitions as follows.

Definition 5: [46, Section 2] A *stable partition* \mathbb{P} for a roommate market is a permutation Π of the set \mathcal{K} such that (i) for every $M_i \in \mathcal{K}$, either $\Pi(M_i) = \Pi^{-1}(M_i)$ or M_i prefers $\Pi(M_i)$ to $\Pi^{-1}(M_i)$, (ii) if M_i prefers M_j to $\Pi^{-1}(M_i)$ then M_j prefers $\Pi^{-1}(M_j)$ to M_i . We refer to $\Pi(M_i)$ and $\Pi^{-1}(M_i)$ as the successor and predecessor of M_i , respectively, relative to Π . We refer to a cycle in Π of odd (or even) length as an odd (or even) party.

Note that the stable partition is in fact a permutation instead of a matching structure that is stable [47], [48].

In the rest of this paper, we will first establish the condition for which a stable matching exists in our IO-CA market. We will then develop an algorithm that achieves a stable matching structure if it exists. Otherwise, the proposed algorithm results in a stable partition among MNOs.

As mentioned previously, in practical systems, each MNO can decide to join or leave the IO-CA market under different situations. If an MNO that is not in the current IO-CA market applies to join the market due to the increasing of the traffic in its network, it needs to go through a procedure, referred to as *addition operation*, to decide its potential IO-CA pairing partner before it starts to share the spectrum.

Let us present the detailed operation as follows.

Operation 1: Addition

Suppose an MNO $M_i \notin \mathcal{K}$ tries to join the IO-CA market.

- i) M_i broadcasts the pairing request and price information to MNOs in \mathcal{K} . Each MNO $M_j \in \mathcal{K}$ then evaluates the resulting payoff when forming an IO-CA pair with M_i . Each $M_j \in \mathcal{K}$ inserts M_i into its own preference list and then feedbacks a confirmation message to M_i . M_i can use this received feedback message to evaluate the performance and establish its preference over all MNOs in \mathcal{K} . All MNOs update $\mathcal{K} = \mathcal{K} \cup \{M_i\}$.

- ii) M_i then sends the IO-CA pairing request to its most preferred MNO in \mathcal{K} . If the request sent by M_i is rejected, M_i sends a request to the next most preferred MNO in its preference list. This process is repeated until M_i has been matched with an MNO $M_j \in \mathcal{K}$ or been rejected by all MNOs in \mathcal{K} . There are four possible results of the above process:
 - a) If the request sent by M_i has been rejected by all MNOs, then M_i will not form any matching pair with MNOs in \mathcal{K} .
 - b) If the request sent by M_i has been accepted by an MNO M_j and M_j has not sent the IO-CA pairing request to another MNO before, it will have the following two results:
 - b-i) M_j is currently not matched with any other MNO, then M_i and M_j will form a matching pair with each other,
 - b-ii) If the request sent by M_i has been accepted by an MNO M_j that is currently matched with another MNO M_k , then M_i and M_j will form a matching pair with each other. M_k will repeat the same Step ii) as M_i ,
 - c) If the request sent by M_i has been accepted by an MNO M_j who has sent an IO-CA pairing request to another MNO before, it means that both M_i and M_j belong to a cycle sequence and there is no stable matching for these two MNOs.
-

Let us consider the case that the MNOs sequentially join the IO-CA market using the above operation. More specifically, at the beginning of the IO-CA market, there is only one MNO (e.g., M_i) in the market. When the second MNO M_j joins the market for $M_j \neq M_i$, its preference list only consists of two elements M_i and M_j and if $P_j(M_i) < P_j(M_j)$, M_j will send an IO-CA request to MNO M_i and a stable matching pair can only be formed when M_i also observes $P_i(M_j) < P_i(M_i)$. If a third MNO M_k tries to join the market for $M_k \notin \{M_i, M_j\}$, it will sequentially send pairing requests to the MNOs in its preference list. If M_i is the first MNO that accepts the request of M_k , it means that M_k is more preferred by M_i than M_j , i.e., $P_i(M_k) < P_i(M_j) < P_i(M_i)$. Since MNO M_k sends requests to MNOs according to its preference list, M_i is also the most preferred MNO that accepts the request of M_k . In this case, M_i and M_k will be paired with each other and M_j will be left without any IO-CA pairing partner. When a fourth MNO M_l tries to join the market for $M_l \notin \{M_i, M_j, M_k\}$, M_l will send a pairing request according to its preference list which will result in the following possible cases: 1) if the request of M_l has been rejected by all three MNOs in the market, M_l will not be paired with any MNO, 2) if MNO M_j is the first MNO that accepts the request of M_l , then an IO-CA pair will be formed by M_l and M_j , 3) if MNO M_i (or M_k) accepts its request, an IO-CA pair will be formed by M_l and M_i (or M_k) and M_k (or M_i) will again restart the requesting process by sending an IO-CA request to its next preferred MNO. If during the requesting process of M_k , M_j is the first MNO to accept the request, then the resulting IO-CA market will consist of two stable matching pairs $\langle M_l, M_i \rangle$ and $\langle M_j, M_k \rangle$. However, it is also possible that M_l will be the first MNO that accepts the request of M_k (For example, during the previous requesting process of M_l , M_l prefers M_k to M_i . However, M_k rejects the request sent by M_l because M_k prefers M_i to M_l). This means that M_l, M_k and M_i form a cyclic sequence and the requesting process will be infinitely repeated among

MNOs M_l, M_k and M_i . In this case, no stable matching exists. We therefore can have the following results.

Proposition 2: Suppose all MNOs sequentially join the IO-CA market following the procedure described in the addition operation. The resulting structure is a stable matching if the IO-CA market consists of a stable matching. Otherwise, the resulting structure will be a stable partition.

Proof: See Appendix C. ■

We can prove the following complexity results about the addition operation.

Proposition 3: The complexity of the addition operation is $O(K^2)$ in the worst case, where K is the number of MNOs³.

Proof: See Appendix D. ■

Similarly, if an MNO that is currently in the market decides to retrieve its exclusive spectrum and quit the IO-CA market due to the decrease of the traffic in its exclusive spectrum, it also needs to inform all other MNOs that its spectrum will no longer be available. Let us present the deletion operation as follows.

Operation 2: Deletion

Suppose an MNO $M_i \in \mathcal{K}$ decides to leave the IO-CA market. Then we have

- i) M_i broadcasts a leaving message to MNOs in $\mathcal{K} \setminus \{M_i\}$ and then each MNO $M_j \in \mathcal{K} \setminus \{M_i\}$ will remove M_i from its preference list and update $\mathcal{K} = \mathcal{K} \setminus \{M_i\}$.
 - ii) If M_i is currently in an IO-CA pair with $M_j = \Gamma(M_i)$ and $M_i \neq M_j$, M_j will send requests to the remainder of the MNOs in \mathcal{K} following the exactly the same line as steps ii) in the addition operation.
-

Following the same line as Proposition 2, we can prove the following results.

Proposition 4: Suppose an MNOs has been deleted from the IO-CA market following the procedure described in deletion operation. The resulting structure is a stable partition.

Proof: See Appendix E. ■

Let us now present the following algorithm that can jointly optimize the transmit powers, pricing coefficients and pairing of MNOs.

Algorithm 1: A Joint Optimization Algorithm

Initialization: Let \mathcal{P}_i be the preference list of M_i and \mathcal{R}_i be the domain of \mathcal{P}_i .

Phase I — Price Adjustment and Power Control

WHILE $\exists M_i \in \mathcal{K}$, $|\mathcal{R}_i| \leq K - 1$,

- 1) Each MNO M_i randomly chooses another MNO $M_j \notin \mathcal{R}_i$ to form an IO-CA pair.
- 2) Once an IO-CA has been formed, the pairing MNOs inform each other regarding their sets of sub-bands allowing aggregation. Each MNO also sends a short training signal in each of these sub-bands for the other pairing MNO to estimate the channel gain between the licensed subscribers and aggregators as well as the transmit powers of the subscribers in each of these sub-bands.
- 3) Both of the pairing MNOs inform each other of their optimal pricing coefficients calculated by Theorem 1 and each aggregator transmits using the transmit power calculated by (13).
- 4) Each MNO M_i obtains the resulting payoff ϖ_{ij} and updates $\mathcal{R}_i = \mathcal{R}_i \cup \{M_j\}$. M_i also updates the preference list by

³In this paper, we follow Bachmann-Landau notations: $f = O(g)$ if $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} < +\infty$.

ranking all MNOs in the updated \mathcal{R}_i from the highest to the lowest payoffs.

ENDWHILE

Phase II — IO-CA Pairing

WHILE $\exists M_i \in \mathcal{K}$ who did not receive any pairing request or did not send any pairing request to other MNOs,

- 5) Each MNO $M_i \in \mathcal{K}$ waits for a bounded random amount of time before using the addition operation to join the IO-CA market.
 - 6) Whenever an MNO M_l wishes to join or leave the market, it uses the addition or deletion operation to join or leave the market.
-

Proposition 5: Algorithm 1 either reports no stable matching exists and achieves a stable partition or generates a stable matching for the IO-CA system. For any IO-CA pair between MNOs M_i and M_j for $M_i \neq M_j$, the transmit power of each aggregator achieves the optimal transmit power derived in Section IV-A. The pricing coefficient β_i and the transmit power \hat{w}_{ji} in the spectrum of each MNO M_i achieve the Stackelberg equilibrium.

Proof: The second part of the above theorem directly comes from (13) and the results of Theorem 1. In the Phase II of Algorithm 1, each MNO enters or leaves the IO-CA market by using the addition and deletion operators introduced in Operations 1 and 2, respectively. Following the results in Propositions 2 and 4, the first part of Proposition 5 can be proved. ■

From the above proposition, if Algorithm 1 reports a stable matching structure, we can claim the existence of at least one stable matching structure. However, if a stable matching does not exist, then Algorithm 1 will result in a stable partition. Note that a stable partition is actually a permutation and is not necessarily stable because of the existence of odd parties. It has been proved in [49] that for each odd party, if an MNO can be removed from this odd party, the rest of the MNOs can form a stable matching with each other. In other words, a possible solution to reach a stable structure among MNOs can be obtained by choosing an MNO from each odd party and forcing it to quit the market. However, which MNO should quit and how to design a distributed mechanism to incentivize the quitting process of these MNOs is out of the scope of the current paper.

It can be observed from Algorithm 1 that the resulting matching structure among MNOs is closely related to the preference relation of each MNO, which also depends on the resulting payoff, by forming different IO-CA pairs with each other. In addition, from the discussion of Section IV-A, we can observe that the resulting payoffs as well as the preference list of each MNO, are directly determined by their transmit powers and pricing coefficients. By using the optimal transmit power in (13) and optimal pricing coefficients derived in Theorem 1, each MNO can obtain the highest payoff when forming an IO-CA with another MNO in the market and hence each MNO cannot further improve its payoff by unilaterally changing its price, transmit power or pairing partner.

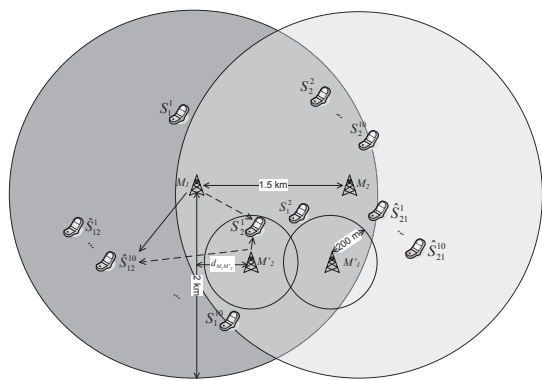


Fig. 3. Simulation setup with two MNOs and multiple UEs.

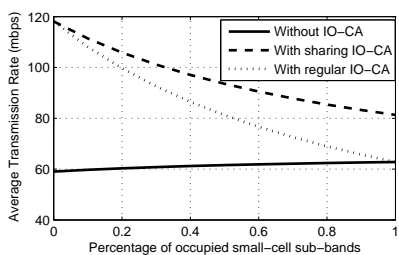


Fig. 4. Average transmission rate for each subscriber with regular IO-CA, sharing IO-CA and without IO-CA.

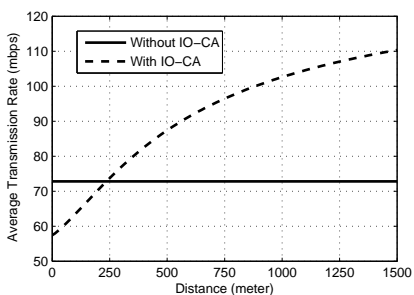


Fig. 5. Average transmission rate for each subscriber with different distances between the macro-cell base station and small-cell base station.

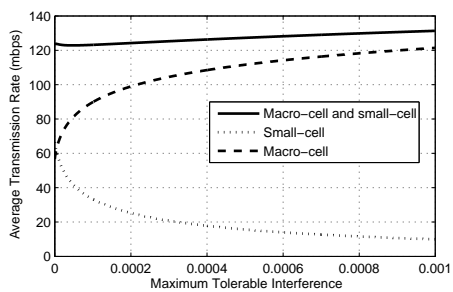


Fig. 6. Average transmission rate for each subscriber with different maximum tolerable interference of small-cell subscribers.

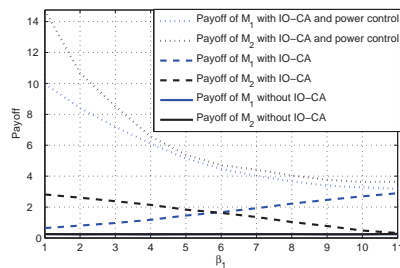


Fig. 7. Payoffs of MNOs with different pricing coefficients: Each MNO consists of 10 subscribers and 10 aggregators and applies the same pricing coefficient to all sub-bands. Pricing coefficient β_2 of MNO M_2 has been fixed to $\beta_2 = 6$.

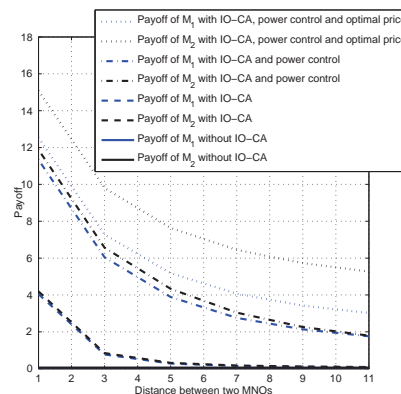


Fig. 8. Payoffs of MNOs under different $d_{M_1 M_2}$, measured in meters.

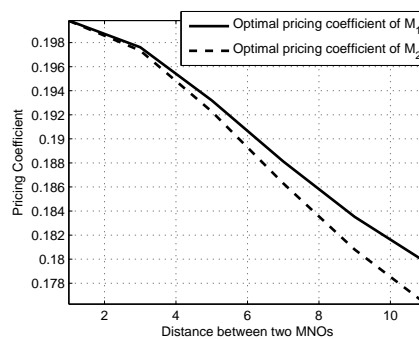


Fig. 9. Optimal pricing coefficients for two subscribers S_1^1 and S_2^1 with different $d_{M_1 M_2}$, measured in meters.

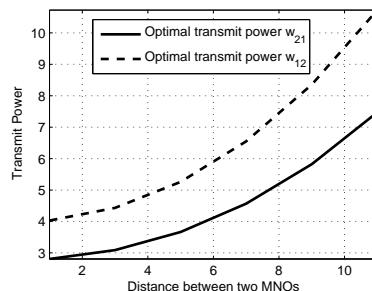


Fig. 10. Optimal transmit powers of two aggregators \hat{S}_{21}^1 and \hat{S}_{12}^1 with different $d_{M_1 M_2}$, measured in meters.

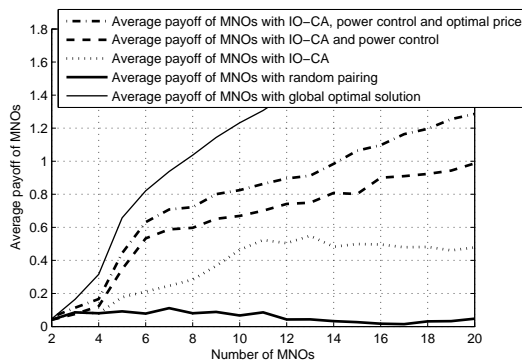


Fig. 11. Average payoff of MNOs with different numbers of MNOs.

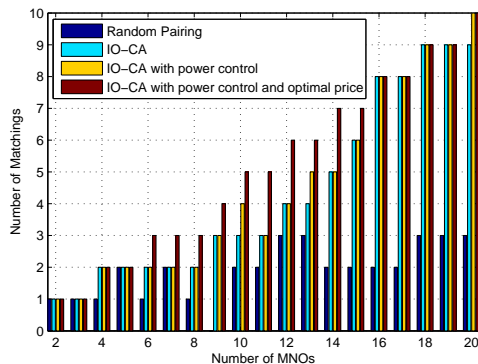


Fig. 12. Number of IO-CA pairs with different numbers of MNOs.

V. DISCUSSIONS AND NUMERICAL RESULTS

Before presenting the simulation results of our proposed joint optimization framework, let us first verify the performance improvement measured by transmission rate brought by the IO-CA in a two-tier heterogeneous network with two closely located MNOs M_1 and M_2 , i.e., we apply the utility function given in (8) with $\alpha_j^k = 1$ and $\beta_j^k = 0 \forall k \in \{1, 2, \dots, K\}$ and $j \in \mathcal{L}$. We assume the network of each MNO consists of a macro-cell overlaid with a small-cell, and that the macro-cells and small-cells

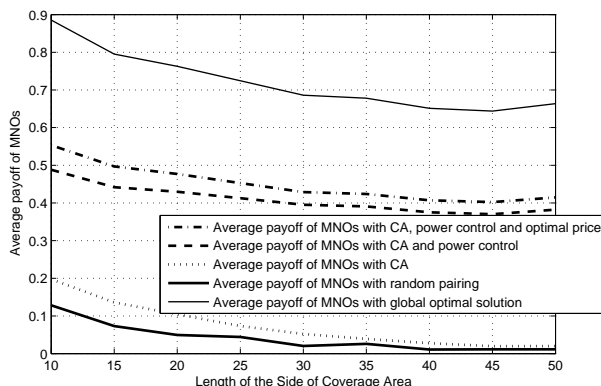


Fig. 13. Average payoff of MNOs with different lengths of the side of coverage area.

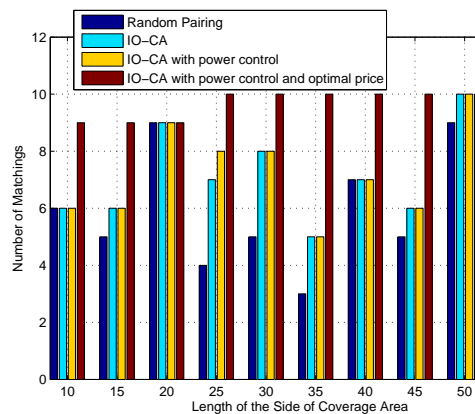


Fig. 14. Number of IO-CA pairs with different lengths of the side of coverage area.

associated with the same MNO operate in different spectrum. We consider the downlink transmission and assume only the macro-cell of each MNO can share the spectrum licensed to the small-cell of the other MNO. With slight abuse of notation, we denote the macro-cell BS and small-cell BS for each MNO M_i by M_i and M'_i , respectively for $i \in \{1, 2\}$. We assume the locations of the macro-cell BSs and small-cell BSs for the two MNOs are symmetric as shown in Figure 3. We assume each (macro- or small-) BS has been associated with the same number of subscribers which are uniformly randomly distributed within the circular coverage area of its BS with radius of 2 km and 200 m for macro- and small-cell, respectively. We set the maximum transmit powers for macro-cell BS and micro-cell BS as 40 and 20 dBm, respectively [50], and assume each BS can adjust its optimal transmit power using (13) when possible.

We first present the average transmission rates of each subscriber achieved by regular IO-CA and sharing IO-CA and compare it with the system without IO-CA in Figure 4. It can be observed that if all the sub-bands in the small-cell are vacant, sharing IO-CA and regular IO-CA will result in the same performance. However, if all the sub-bands are fully occupied by small-cell subscribers, regular IO-CA cannot provide any performance improvement compared to the system without IO-CA. We can also observe that the sharing IO-CA can always improve the transmission rate of the subscribers even when there is no vacant small-cell sub-band available. In Figure 5, we compare the average transmission rate of the subscriber for each MNO with and without sharing IO-CA under different distances $d_{M_i M'_j}$. It can be easily observed that when $d_{M_i M'_j}$ approaches zero, our sharing IO-CA can be regarded as special case of the traditional carrier aggregation between a small-cell and macro-cell of the same MNO sharing the same spectrum. We can observe from Figure 5 that if $d_{M_i M'_j}$ is close to zero, IO-CA cannot improve the transmission rate for its subscriber compared to the case without IO-CA. This is because the high-power macro-cell subscribers and the

low-power small-cell subscribers can cause large cross-interference when they are closely located within the same macro-cell. However, with the increasing of the distance between the macro-cell BS of one MNO and small-cell BS of the other MNO, the transmission rate for each subscriber can be significantly improved. This confirms our previous observation that IO-CA has the potential to significantly improve the performance of cellular networks compared to carrier aggregation within a single MNO's network. In sharing IO-CA, the aggregators from one MNO should always control their transmit powers to avoid intolerable interference to the subscribers in the small cell of the other MNO. Therefore, in Figure 6, we assume the small cell subscribers have the same maximum tolerable interference levels and each macro-cell subscriber adapts its transmit power to the maximum tolerable interference level of its sub-band sharing small-cell subscriber. We compare the average transmission rate for our simulated network system under different tolerable interference levels. We can observe that the transmission rates of the small-cell subscribers decrease with the maximum tolerable levels. On the other hand, the increasing of the transmission rate for the aggregators from the other MNO can compensate the performance degradation of the small-cell subscribers and the total average transmission rate for each subscriber can increase with the maximum tolerable interference level.

In Figures 4-6, we assume all subscribers are uniformly randomly located within a fixed coverage area and compare the performance of cellular networks with and without IO-CA. The performance of each subscriber is also closely related to its relative distance to the corresponding BS. We will provide a more detailed discussion about this in examining Figures 13 and 14 at the end of this section.

In this paper, we consider the joint optimization of three problems: pairing problem, pricing adjustment problem and power control problem. We derive solutions for each of these problems and propose a joint optimization algorithm that simultaneously achieves all these solutions. Our algorithm is general in the sense that each separate part of our algorithm can be individually applied to optimize IO-CA-based cellular networks under different situations. For example, if each aggregator cannot keep track of the channel gains between itself and the subscribers, it will fix its transmit power. However, MNOs can still use Phase-II of Algorithm 1 to decide their IO-CA pairing partners. In the rest of section, we present numerical results to access the performance of our proposed optimization algorithms. We mainly compare the following approaches for our IO-CA-based cellular networks,

- 1) *Random pairing*: all K MNOs are randomly partitioned into $\lceil \frac{K}{2} \rceil$ groups each of which consists of two MNOs. If both of MNOs in a group can improve their payoffs using IO-CA with predefined fixed powers and prices, they will form an IO-CA pair. Otherwise, both MNOs will only use their own

licensed spectrum to transmit signals without aggregating the spectrum of each other.

- 2) *IO-CA*: each MNO fixes the powers and pricing coefficient and only uses Phase-II of Algorithm 1 to decide its IO-CA pairing partner.
- 3) *IO-CA with power control*: each MNO uses the optimal transmit power calculated from (13) and Phase-II of Algorithm 1 to decide its IO-CA pairing partner.
- 4) *IO-CA with power control and optimal price*: each MNO uses Algorithm 1 to decide its transmit power, pricing coefficient and the pairing partner.

From the discussion in Section III, we can observe that in regular IO-CA, there is no incentive for each MNO to control the transmit powers of the aggregators by optimizing its price. In addition, regular IO-CA can be regarded as a special case for sharing IO-CA when the interference caused by the sub-band sharing aggregator is lower than the maximum tolerable interference of the subscriber even when the aggregator uses its maximum transmit power, i.e., $\tilde{q}_{ij}^k \leq \frac{q_i^k}{h_{ij}^k}$. Therefore, in this section, we mainly focus on the sharing IO-CA. We first simulate IO-CA-based cellular network with two MNOs, each of which corresponds to a cellular network with a base station located at the center of the coverage area. Each MNO also contains a set of subscribers and a set of aggregators uniformly and randomly located in the overlapped coverage area of both MNOs as is shown at the top of Figure 3. Each subscriber or aggregator corresponds to the uplink communication channel from each UE to the base station. Assume that the channel gain h_{ij}^k is given by $h_{ij}^k = \frac{\hat{h}_{ij}^k}{d_{ij}^k \xi}$ for $i, j \in \{1, 2\}$ where \hat{h}_{ij}^k is the average channel fading coefficient, d_{ij}^k is the distance between aggregator \hat{S}_{ij}^k and the base station of M_j and ξ is the fading exponent. We also use $d_{M_1 M_2}$ to denote the distances between base stations of M_1 and M_2 . Let us focus on the performance of both source-to-destination pairs with different values of $d_{M_1 M_2}$.

We first consider the effects of the changing pricing coefficients on the payoff of the MNOs. In Figure 7, we assume each MNO applies the same pricing coefficient to all of its sub-bands. We then fix the pricing coefficient of one MNO and compare the payoffs of MNOs when the other MNO changes its pricing coefficient. It is observed that the payoff of both MNOs will be affected even when the price of only one MNO changes. This is because in our model each MNO can use the price to control the payoff obtained from its own spectrum as well as that obtained from aggregating the spectrum of the other MNO. Another observation is that the IO-CA with power control significantly increases the payoff of both MNOs, and more importantly, it also reduces the payoff difference between the MNOs caused by the price change.

The payoffs of both MNOs with different optimization algorithms under different values of $d_{M_1 M_2}$ are compared in Figure 8. It is observed that the payoffs of both MNOs decrease with the distance $d_{M_1 M_2}$. This is because when

the distance between MNOs becomes large, each MNO will decrease its pricing coefficient to attract more aggregators from the other MNO which also reduces the revenue obtained from the other MNO. We can also observe that the payoff improvement brought by IO-CA with power control and/or optimal price is larger than those brought by other two approaches.

To study the payoff obtained by MNOs from each subscriber, we present the optimal pricing coefficient of each MNO in one of its sub-bands occupied by subscribers S_1^2 and S_2^1 under different $d_{M_1M_2}$ in Figure 9. We observe that the optimal pricing coefficients charged in both sub-bands of MNOs M_1 and M_2 decrease with the distance $d_{M_1M_2}$. This is because when two MNOs are further away, each MNO will need to provide more incentive such as reducing its price to attract the other MNO to aggregate its spectrum.

In Figure 10, we compare the optimal transmit powers of two aggregators randomly chosen for both MNOs under different distances between the base stations. We can observe that with the increasing of $d_{M_1M_2}$, the aggregator should always increase its transmit power to further improve the payoff of its corresponding MNO because the cross-interference for each sub-band sharing subscriber and aggregator decreases with $d_{M_1M_2}$. Note that the price decreasing process illustrated in Figure 9 affects the revenue for MNOs at a much faster rate, which eventually lowers the payoffs of the MNOs as observed in Figure 8.

We now simulate an IO-CA-based cellular network with more than two MNOs by considering a square-shaped coverage area in which each MNO has a fixed number of subscribers and aggregators uniformly randomly located in the coverage area. We follow the same settings as the two MNO case introduced in the beginning of this section.

In Figure 11, we fix the size of the coverage area and compare the average payoff obtained by all MNOs under different total numbers of MNOs. We can observe that random pairing cannot provide any payoff improvement to MNOs because the chance for each MNO to pick up a high cross interfering pairing partner (e.g., another MNO that is close-by) increases with the density of the MNOs in the coverage area. However, the average payoff of MNOs increases with the number of MNOs when IO-CA is allowed. This is because all MNOs are randomly located in the area and with the increasing of the number of MNOs, each MNO will have more choice of its IO-CA pairing partner using the Phase II of Algorithm 1. We can also observe that as the coverage area becomes more and more crowded, the payoff improvement brought by our proposed IO-CA with power control and optimal price becomes more significant. In other words, our proposed joint optimization algorithm is more useful in a high population/MNO density area such as city center or during the peak hours of the data service demand. Note that, in our model, we assume MNOs are selfish and we focus on the distributed optimization for cellular networks

with multiple MNOs. In our setting, an IO-CA pair can only be formed if both pairing MNOs can further improve their performance by allowing their spectrum to be aggregated by each other. This condition is referred to as *individual rationality* in game theory. It can be observed that the payoff sum of the MNOs can be further increased if some MNOs sacrifice their performance and allow other MNOs to aggregate their spectrum at a low price. We refer to the solution that can maximize the total payoff sum of all MNOs without the constraint of individual rationality as the global optimal solution which is also presented in Figure 12. As can be observed from Figure 11, although the global optimal solution is significantly better than our proposed distributed optimization approach, it cannot guarantee stability and the performance for each individual MNO, and hence cannot always incentivize IO-CA among MNOs.

In Figure 12, we compare the numbers of IO-CA pairs formed under different number of MNOs. It can be observed that the number of IO-CA pairs between MNOs achieved by random pairing does not vary much as the number of MNOs increases. However, if the power control and/or optimal prices have been applied, the chance for each MNO to find another MNO to form an IO-CA pair will increase with the number of MNOs. In addition, when the number of MNOs is large enough (e.g., exceeds 16 in Figure 12), IO-CA with power control can achieve the maximum number of IO-CA pairs among MNOs. In other words, if the main objective for each MNO that adopt IO-CA is to maximize the total number of spectrum sharing pairs, IO-CA with power control and IO-CA with power control and optimal price achieve the same results if the density of MNOs in the coverage area exceeds a certain threshold.

In Figure 13, we compare the payoffs of the MNOs under different sized coverage areas. We observe that the average payoff of MNOs decreases when the size of the coverage area becomes large. This verifies our previous observation that our proposed optimization algorithm can provide high performance improvement when the density of the MNOs is high. We also observe that the average payoff obtained only by IO-CA approaches that obtained by random pairing when the length of the coverage area becomes large. However, IO-CA with optimal price and/or power control can still provide significant payoff improvement compared to the random pairing.

In Figure 14, we compare the number of IO-CA pairs between MNOs under different sizes of the coverage area. We observe that if the size of the network is small, there are always some MNOs that cannot find a pairing partner to form an IO-CA pair. However, when applying IO-CA with optimal price and power control, the number of IO-CA pairs between MNOs will reach the maximum number $\frac{K}{2}$ when the size of the network becomes large.

VI. CONCLUSION

This paper considers CA between MNOs in a cellular network. In this network, an MNO can not only access its

own licensed spectrum, but can also aggregate the spectrum licensed to other MNOs by paying a certain price. We establish a stable roommate market to study the pairing problem among the MNOs. We derive a condition for which a stable matching structure exists. We propose an algorithm to approach a stable matching structure if it exists. Otherwise, the algorithm results in a stable partition. We then establish a Stackelberg game-based model to study the interaction between the subscribers and aggregators in the spectrum of each MNO. We derive the optimal transmit power for each aggregator and the Stackelberg equilibrium for each MNO. We propose a joint optimization algorithm that can achieve a stable matching structure among MNOs if it exists as well as the optimal transmit powers and prices for each MNO. We present numerical results to verify the performance improvement brought by each of these optimization methods under different situations.

APPENDIX A PROOF OF PROPOSITION 1

Let us consider the payoff of subscriber \hat{S}_{ij}^k of MNO M_i obtained by aggregating the k th sub-band of MNO M_j as follows,

$$\varpi_{(i)j}^k = B_j^k \log \left(1 + \frac{h_{ii}^k \hat{w}_{ij}^k}{1 + h_{ji}^k w_j^k} \right) - \beta_j^k h_{ij}^k \hat{w}_{ij}^k. \quad (19)$$

It is observed that \hat{w}_{ij}^k increases (or decreases) with $\varpi_{(i)j}^k$ when $\hat{w}_{ij}^k < \hat{w}_{ij}^{k*}$ (or $\hat{w}_{ij}^k \geq \hat{w}_{ij}^{k*}$) where w_{ij}^{k*} is the optimal solution of $\varpi_{(i)j}^k$ given in (13). Because $\varpi_{(i)j}^k = 0$ if $\hat{w}_{ij}^k = 0$, we hence have $\varpi_{(i)j}^k \geq 0$ when $\hat{w}_{ij}^{k*} \geq 0$.

APPENDIX B PROOF OF THEOREM 1

Let us consider the optimization of the pricing coefficients for the MNOs. Before the derivation of the optimal price, we first need to calculate the range of β_i^k . It is observed that the value of β_i^k is limited by two constraints. The first one is the power constraints in (2). The other one is the fact that the transmit power of each aggregator should be a positive value. Otherwise M_i cannot obtain any benefits by optimizing β_i^k in the k th sub-band, i.e., $\hat{w}_{ji}^{k*} > 0$. Substituting \hat{w}_{ji}^{k*} in (13) into (2), we can obtain the lower bound β_i^{k-} for β_i^k which is given in (15). Similarly, applying $\hat{w}_{ji}^{k*} > 0$, we can calculate the upper bound β_i^{k+} of β_i^k which is presented in the results of Theorem 1. In other words, if the value of β_i^{k*} is less than that of β_i^{k-} , the transmit power of aggregator \hat{S}_{ji}^k will exceed the maximum tolerable interference level of M_i . While if β_i^{k*} is greater than β_i^{k+} , \hat{S}_{ji}^k will not aggregate the k th sub-band of M_i by transmitting with a positive power and hence MNO M_i cannot obtain any revenue from the k th sub-band of MNO M_j .

If we substitute \hat{w}_{ij}^{k*} in (13) into the payoff function of (8), we can find that $\varpi_{i(j)}^k(w_i^k, \hat{w}_{ji}^{k*}, \beta_i^k)$ (or $\varpi_{(i)j}^k(w_j^k, \hat{w}_{ij}^{k*}, \beta_j^k)$) is only related to \hat{w}_{ji}^{k*} (or \hat{w}_{ij}^{k*})

which is controlled by β_i^k (or β_j^k). Since MNO M_i can only decide the value of β_i^k , we assume β_j^k has already been chosen by MNO M_j and hence MNO M_i only needs to focus on the optimization of β_i^{k*} to maximize $\varpi_{i(j)}^k(w_i^k, \hat{w}_{ji}^{k*}, \beta_i^k)$. Let us denote $\rho_i^k = \frac{h_{ji}^k(1+h_{ij}^k w_i^k)}{h_{jj}^k}$ and $\theta_i^k = h_{ii}^k w_i^k$. Substituting \hat{w}_{ij}^{k*} and \hat{w}_{ji}^{k*} in (13) into (8), we have

$$\begin{aligned} & \varpi_{i(j)}^k(w_i^k, \hat{w}_{ji}^{k*}, \beta_i^k) \\ &= B_i^k \log \left(1 + \frac{\beta_i^k \theta_i^k}{B_i^k + \beta_i^k (1 - \rho_i^k)} \right) \\ & \quad + (B_i^k - \beta_i^k \rho_i^k)^+ \end{aligned} \quad (20)$$

To find the optimal value of β_i^k that can maximize $\varpi_{i(j)}^k$, we have

$$\begin{aligned} & \frac{\partial \varpi_{i(j)}^k(w_i^k, \hat{w}_{ji}^{k*}, \beta_i^k)}{\partial \beta_i^k} = 0 \\ & \Rightarrow \frac{B_i^k (1 + \theta_i^k - \rho_i^k)}{B_i^k + \beta_i^k (1 + \theta_i^k - \rho_i^k)} \\ & \quad - \frac{1 - \rho_i^k}{B_i^k + \beta_i^k (1 - \rho_i^k)} - \rho_i^k = 0 \\ & \Rightarrow \frac{\beta_i^k B_i^k \rho_i^k (2 - 2\rho_i^k + \theta_i^k)}{(B_i^k + \beta_i^k (1 + \theta_i^k - \rho_i^k)) (B_i^k + \beta_i^k (1 - \rho_i^k))} \\ & \quad + \frac{B_i^{k2} (\theta_i^k - \rho_i^k) - \beta_i^{k2} \rho_i^k (1 - \rho_i^k) (1 + \theta_i^k - \rho_i^k)}{(B_i^k + \beta_i^k (1 + \theta_i^k - \rho_i^k)) (B_i^k + \beta_i^k (1 - \rho_i^k))} \\ & \quad = 0. \end{aligned} \quad (21)$$

From the above equation, it is observed that if $(2 - 2\rho_i^k + \theta_i^k)^2 < \theta_i^k (\theta_i^k - \rho_i^k)$, there is no solution for $\frac{\partial \varpi_{i(j)}^k(w_i^k, \hat{w}_{ji}^{k*}, \beta_i^k)}{\partial \beta_i^k} = 0$. In this case, $\frac{\partial \varpi_{i(j)}^k(w_i^k, \hat{w}_{ji}^{k*}, \beta_i^k)}{\partial \beta_i^k} < 0$ which means that $\varpi_{i(j)}^k(w_i^k, \hat{w}_{ji}^{k*}, \beta_i^k)$ always decreases with β_i^k . Therefore, MNO M_i should choose the lowest value of β_i^k in the k th sub-band to maximize the payoff $\varpi_{i(j)}^k(w_i^k, \hat{w}_{ji}^{k*}, \beta_i^k)$, i.e., $\beta_i^{k*} = \beta_i^{k-}$. However, if $(2 - 2\rho_i^k + \theta_i^k)^2 \geq \theta_i^k (\theta_i^k - \rho_i^k)$, there exist two solutions for the equation (21) which are given in (17) and (18), respectively. These solutions can be the value of β_i^k that either maximizes or minimizes $\varpi_{i(j)}^k(w_i^k, \hat{w}_{ji}^{k*}, \beta_i^k)$. Note that in (16), we also consider the boundary values of β_i^k . This is because the solution β_i^{k*} that maximizes the value of $\varpi_{i(j)}^k(w_i^k, \hat{w}_{ji}^{k*}, \beta_i^k)$ may not always within the range of β_i^k . In this case, we need to choose the lowest or the highest value of β_i^k to improve the payoff of MNO M_i in the k th sub-band.

Note that the pricing coefficients of MNOs control the optimal transmit powers of MNOs. We hence can claim that the optimal pricing coefficient β_i^{k*} and the corresponding transmit powers \hat{w}_{ji}^{k*} achieve an Stackelberg equilibrium for the Stackelberg game of the leader (subscribers) and the follower (aggregators). This concludes our proof.

APPENDIX C
PROOF OF PROPOSITION 2

The proof of the above proposition follows directly from the results proved in [27], [45]–[47], [51], [52]. We list these results as follows:

- R1) [45, Theorem 6.6] For any stable roommate market, there exists at least one stable partition and any two stable partitions have the same odd parties.
- R2) [45], [46, Theorem 6.7] There is no stable matching available for a roommate market if and only if there exists a stable partition with a set that has an odd party,
- R3) Each even party can be broken into pairs of mutually agreed MNOs preserving stability [45],
- R4) If a stable roommate market contains at least one stable matching structure, the set of all stable partitions and the set of all the stable matching coincide [47].

Let us briefly describe how to prove Proposition 2 using the above results. From Step ii) in the addition operation, we can observe that if a new MNO M_i sequentially sends pairing requests to other MNOs in the market from the most preferred MNO to the least preferred one, we can claim that for each MNO M_j that rejects the request of M_i , there must exist another MNO M_k that is strictly preferred by M_j . In addition, any other MNO M_l which is more preferred by M_i than M_j (e.g., satisfying $P_i(M_l) < P_i(M_j)$) has already rejected the requests of M_i . The sequential requests of M_i will have the following possible results:

- 1) If M_i has been rejected by all the MNOs in the market that are more preferred by M_i than matching with itself, it means that either MNO cannot find any MNO that can improve its payoff by forming an IO-CA pair, or every MNO that can provide performance improvement for M_i by forming an IO-CA pair has already been matched with another MNO that is more preferred than M_i . In this case, there is no stable matching for the IO-CA market and M_i will not form an IO-CA pair with any other MNO in the market. This results in the cases in Step ii-a).
- 2) If there is one MNO M_j accepting the request of M_i for $M_j \neq M_i$ and $P_i(M_j) < P_i(M_i)$, it means that M_j prefers M_i to its current matching partner M_l and all the other MNOs that are more preferred by M_i than M_j prefer their current matching partner to M_i . Therefore, both M_i and M_j have the incentive to form an IO-CA pair. Once an IO-CA pair has been formed between M_i and M_j , M_l will start the sequential requesting process as M_i . The same results as described in 1) and 2) will also apply for M_i . It is possible for M_l to also find another MNO M_m that accepts its request and form a matching pair with M_l by separating from its current IO-CA pairing partner. If this process continues, it will result in a sequence of matching, separating and sequential requesting processes of a set of MNOs. There are two possible results. If this sequence of processes

will result in a new stable matching structure in which all MNOs have found their new IO-CA pairing partners, it means that the final matching is stable. This results in the cases in Step ii-b). If an MNO M_n accepting the request of another MNO has previously sent a request, it means that there is a cycle sequence and the MNOs involved in the sequence of processes will repeatedly send requests to each other, forming a pair and separating from their matching partner and it is easy to verify that the sequence of MNOs in the sequence of processes forms an odd party. This results in the cases in Step ii-c). Using the result R2), we can claim that there will be no stable matching for the MNOs in the sequence of processes. Also using result R1), we can claim that the addition operation always results in the same odd parties. Finally, using result R4), we can claim that if there is no odd parity, the final result of the addition operation is always a stable matching.

Based on the above analysis, we hence can claim that if at least one stable matching exists, the addition operation will achieve it. If no stable matching exists for the IO-CA market, the addition operation will result in a stable partition. This concludes the proof.

APPENDIX D
PROOF OF PROPOSITION 3

As can be easily observed from the addition operation, the worst case happens when the new MNO M_i joins the IO-CA market and finds a pairing partner (e.g., MNO M_j) which is in a transposition with another MNO M_n . In this case, MNO M_n will repeat steps ii) again. If the same situation happened repeatedly for each of the other MNOs in \mathcal{K} , this will cause all K MNOs to send requests to each of the other $K - 1$ MNOs and hence results in a complexity of $O(K^2)$ in the worst case.

APPENDIX E
PROOF OF PROPOSITION 4

Let us use the following result and Proposition 2 to prove Proposition E.

- R5) Suppose Π is a stable partition in a stable roommate market and $\mathcal{C} = \langle a_{i_1}, a_{i_2}, \dots, a_{i_{2k+1}} \rangle$ is an odd party in Π for $k \geq 1$. Then $\Pi' = (\Pi \setminus \mathcal{C}) \cup \langle a_{i_1}, a_{i_2} \rangle \langle a_{i_3}, a_{i_4}, \dots, \langle a_{i_{2k-1}}, a_{i_{2k}} \rangle \rangle$ is a stable partition of $\Pi \setminus \langle a_{i_{2k+1}} \rangle$ [45], [49].

From the above result, we can claim that if MNO M_i has been deleted from the IO-CA market and M_i belongs to an odd party, each of the remaining MNOs in the same odd party as M_i will be able to find its IO-CA pairing partner and form a stable matching pair.

If M_i has already been matched to another MNO M_j for $M_j \neq M_i$, then M_j will separate with M_i and find its pairing partner using the same procedure as the addition operation. Therefore, we can use Proposition C to prove that the resulting matching will be a stable partition. This concludes the proof.

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